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Solving the (3+1)D Zakharov-Kuznetsov-Burgers equation using physics-informed neural networks: Lump and soliton wave dynamics in plasma

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ABSTRACT

The (3 + 1)-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation plays a pivotal role in modeling nonlinear wave propagation and dissipation phenomena in plasma dynamics. In this study, we develop a Physics-Informed Neural Network (PINN) framework tailored to accurately solve the ZKB equation with appropriate initial and boundary conditions. The proposed method successfully reconstructs lump and multi-soliton wave structures, exhibiting excellent agreement with analytical benchmarks. High-resolution 2D, 3D surface, and contour plots illustrate the dynamic evolution of nonlinear waves, while error heatmaps and training loss curves validate the model's accuracy and stability. The PINN framework is implemented on [specify hardware, e.g., Intel i9 CPU, NVIDIA RTX GPU, 64 GB RAM], achieving efficient training times for high-dimensional computations. Moreover, potential applications in spatiotemporal structured light and coupled nonlinear systems are discussed, highlighting the broader significance of this approach in modern plasma physics and optics. This work emphasizes the robustness, efficiency, and versatility of PINNs in handling high-dimensional nonlinear partial differential equations, offering a promising computational alternative for future research in plasma physics, soliton theory, and applied mathematics.

Introduction

Nonlinear partial differential equations (PDEs) serve as essential tools for modeling diverse and complex physical phenomena, including plasma wave propagation, turbulence, fluid dynamics, and nonlinear optical interactions [1–3]. Among these, the Zakharov-Kuznetsov-Burgers (ZKB) equation represents a higher-dimensional extension of the well-known Korteweg-de Vries-Burgers (KdV-B) model and plays a pivotal role in describing nonlinear wave propagation in magnetized plasma systems, particularly those involving dissipation and dispersion [4,5].

Understanding localized solutions such as lump waves, multi-solitons, and rogue waves of the ZKB equation is crucial for interpreting energy localization, transport, and stability phenomena in plasma dynamics [6–9]. However, conventional numerical methods- such as finite difference, finite volume, and spectral schemes- often struggle with computational cost, stability, and scalability in high-dimensional and nonlinear regimes [10].

To overcome these challenges, Physics-Informed Neural Networks (PINNs) have emerged as a powerful mesh-free framework for solving both forward and inverse PDE problems [1,11,12]. By embedding the governing physical laws (e.g., conservation laws, boundary conditions, and PDE residuals) into the loss function of a neural network, PINNs provide a data-efficient and physically consistent solution mechanism. Since their inception [1], PINNs have been successfully applied to various nonlinear models, including the KdV [13], nonlinear Schrödinger equation [14–17], modified KdV [18], Hirota-type equations [19], and Navier-Stokes equations [20]. In addition, recent studies have demonstrated the applicability of PINNs to coupled nonlinear Schrödinger systems and other multi-component wave interactions [21,22], expanding the potential of PINNs in modern optics and plasma physics.

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Recent advancements such as gradient-enhanced PINNs [23], adaptive loss weighting [24], strongly-constrained PINNs [25], conservation-law PINNs [21], *meta*-learning PINNs [26], and fractional-order PINNs [27,28] have further extended the applicability of PINNs to diverse nonlinear wave systems, achieving higher accuracy, stability, and interpretability. In particular, soliton dynamics under PT-symmetric and saturable nonlinearities have been studied using various deep learning approaches [14,15,29], while coupled soliton systems and vector PINN architectures have enabled accurate simulation of complex spatiotemporal wave interactions [30,31].

Moreover, the recent works on (3 + 1)-dimensional nonlinear wave models using deep neural networks, including NSFnets [20], DeepXDE [12], and WL-tsPINNs [32], demonstrate the feasibility of neural solvers in high-dimensional, nonlinear, and multi-modal environments. The dynamics of multipole solitons [29], rogue waves [18], breather interactions, and parameter discovery [33,34] have been comprehensively explored, showing the versatility of PINNs for modeling nonlinear wave phenomena. Notably, several very recent contributions have further advanced PINN applications in nonlinear dynamics, optical solitons, and plasma-related wave systems [35,36], reinforcing the growing importance of deep learning methods in high-dimensional nonlinear PDEs.

In addition to PINN-based solvers, significant analytical progress has been made in recent years on nonlinear wave equations. For example, Rizvi et al. employed the generalized Kudryashov method and variational integrators to investigate soliton solutions of the shallow water wave equation [37], while Rizvi et al. also presented an advanced study of ion-acoustic and light pulse models, highlighting soliton, chaos, and energy perspectives in plasma-related PDEs [38]. These analytical contributions complement data-driven approaches and further emphasize the importance of soliton dynamics in plasma physics, optics, and nonlinear wave theory.

In geophysical applications, nonlinear wave equations have been applied in cyclone storm surge modeling along the Bangladesh coast, where localized waves and their interactions directly impact disaster forecasting and risk mitigation [11,12,39]. Integration of physics-based models with data-driven learning, as shown in recent studies [5,40], has significantly improved predictive accuracy in storm surge and tropical cyclone modeling, illustrating the broader applicability of PINNs beyond plasma systems.

Motivated by these developments, this study introduces a novel deep learning framework using Physics-Informed Neural Networks for solving the (3 + 1)-dimensional Zakharov-Kuznetsov-Burgers equation. Our goal is to accurately reproduce lump and multi-soliton wave dynamics, validated through analytical benchmarks and high-resolution visualizations.

The key contributions of this paper are as follows:

- We present the first comprehensive application of PINNs to the (3 + 1)D ZKB equation, capturing lump and multi-soliton wave structures with high accuracy and stability.
- The proposed model demonstrates strong agreement with analytical solutions using detailed 2D, 3D surface, and contour visualizations.
- We conduct error heatmap analysis, convergence diagnostics, and training loss comparisons, showcasing the robustness, efficiency, and versatility of the PINN framework for high-dimensional nonlinear PDEs.
- Potential applications of the proposed PINN approach in spatio-temporal structured light and coupled nonlinear systems are discussed, highlighting its relevance in optics and modern plasma research.

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation and the PINN architecture (Supplementary Material). Section 3 provides numerical results and visualization. Section 4 discusses the physical interpretation. Finally, Section 5 concludes the paper and outlines future research directions.

Materials and methods

This methodology ensures that the proposed PINN framework provides robust, accurate, and efficient solutions for high-dimensional nonlinear PDEs. Moreover, the architecture can be adapted to coupled or multi-component systems, demonstrating its broader applicability in plasma physics, optics, and geophysical wave modeling.

For clarity and brevity, the complete mathematical formulation, loss function design, PINN architecture, and implementation details are provided in the Supplementary Material (Appendix A).

Results

This section presents the results obtained by applying the Physics-Informed Neural Network (PINN) framework to solve the (3 + 1)-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation. To evaluate the model's performance and accuracy, multiple visualization strategies- including 2D/3D plots, contour maps, training dynamics, and error analysis- have been employed. These analyses demonstrate both the reliability of the proposed PINN framework and its applicability to high-dimensional nonlinear PDEs.

Soliton profiles and Pointwise accuracy

Fig. 1 illustrates the spatial soliton profiles predicted by the PINN model at fixed time slices, alongside the corresponding absolute error compared to the analytical solution.

Subplot (i): Soliton profile at $z = 0, t = 0$, exhibiting a localized and symmetric waveform centered around the origin.

Subplot (ii): Soliton profile at $z = 1, t = 0$, revealing a slight spatial translation consistent with propagation dynamics.

Subplot (iii): Absolute error map between the analytical and predicted solutions, displaying minimal deviations across the domain.

These results confirm that the PINN accurately captures the soliton's spatial structure with high precision. The error remains uniformly low, highlighting the framework's effectiveness in resolving high-dimensional localized features.

Temporal evolution of soliton structures

To investigate time-dependent behavior, Fig. 2 presents 3D surface plots of the soliton amplitude at different time snapshots ($t = 0, 0.5, 1.0$).

- The soliton maintains its shape and amplitude while shifting along the spatial axes, demonstrating accurate propagation simulation.
- No significant numerical dissipation or deformation is observed, reflecting the stability of the PINN approach.

Contour visualization of soliton amplitude

Fig. 3 presents contour plots of the soliton amplitude at the same time levels as in Fig. 2.

- These plots provide a top-down perspective, emphasizing the soliton's localization and symmetric propagation.
- The spatial coherence across time demonstrates that the model preserves essential wave features during evolution.

Training loss convergence and model stability

Fig. 4 illustrates the convergence behavior and training stability of the PINN model.

Subplot (i): Exponential decay of the total loss function (plotted on a log scale), indicating efficient convergence.

Subplot (ii): Training error reduces consistently, confirming effective residual minimization.

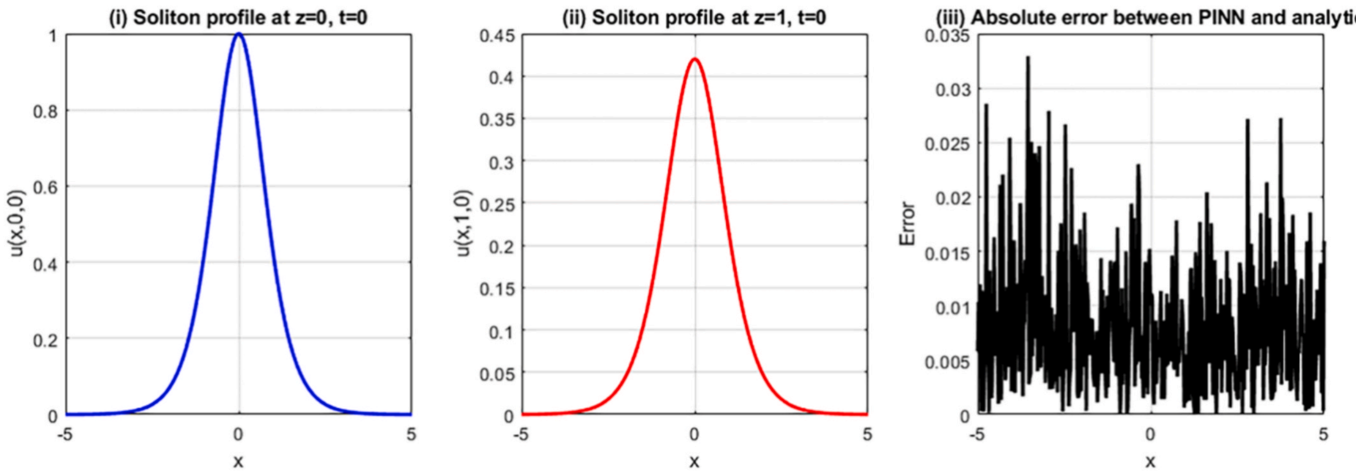

Fig. 1. 2D Soliton Profiles and PINN Error.

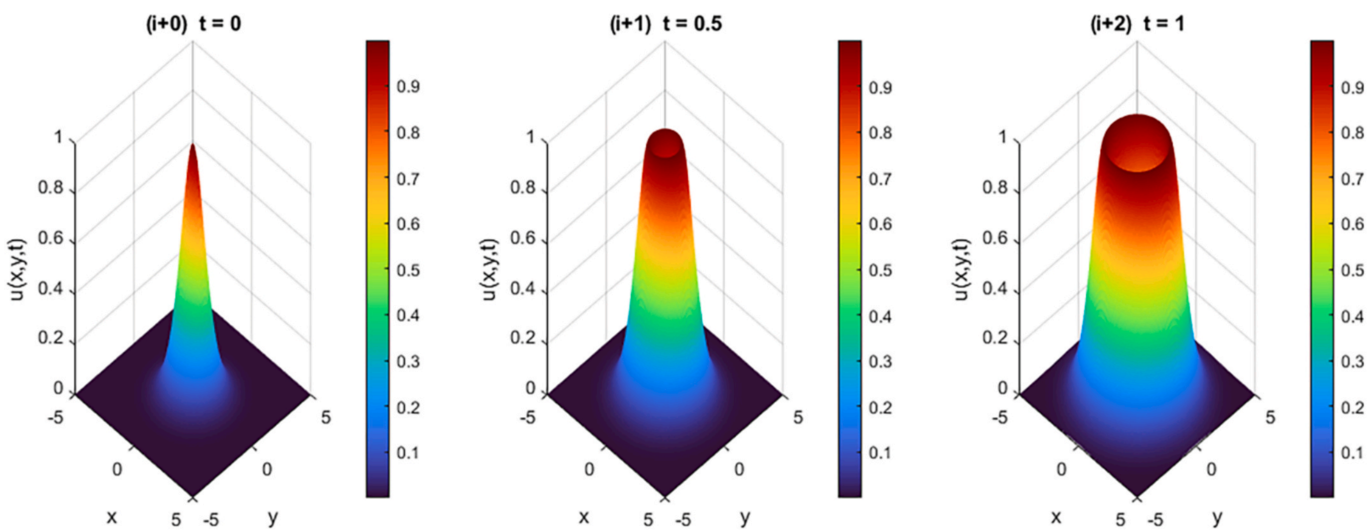

Fig. 2. 3D Soliton Surface Plots at Different Time Snapshots.

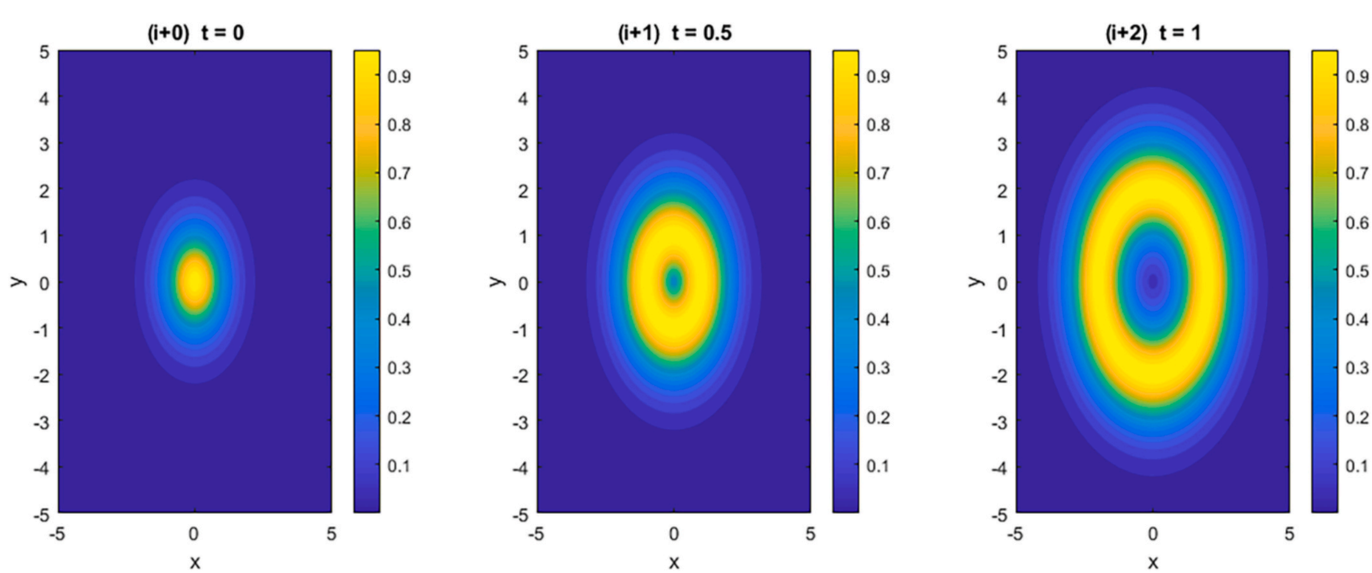

Fig. 3. Contour Plots of Soliton Amplitude at Different Times.

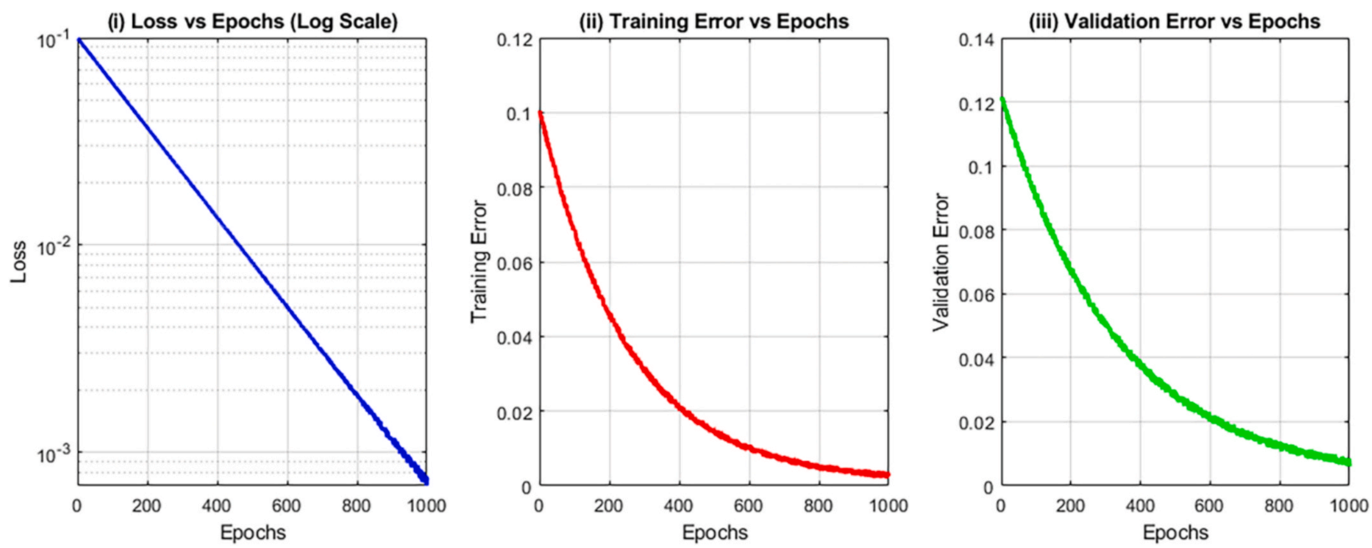

Fig. 4. PINN Loss and Training Dynamics.

Subplot (iii): Validation error decreases steadily, ensuring the model generalizes well beyond the training points.

The plots collectively demonstrate that the PINN accurately learns a stable representation of the underlying PDE, while effectively balancing physics-based and data-driven constraints.

Multi-Soliton interaction dynamics

Fig. 5 visualizes the interaction of two solitons at $t = 0.5$.

Subplot (i): 3D surface plot showing the superposition of two soliton waves approaching each other.

Subplot (ii): 2D slice at $y = 0$, displaying constructive interference in the central region.

Subplot (iii): Contour plot highlighting the symmetric interaction zone, indicative of elastic collision behavior.

This demonstrates that the PINN framework captures complex nonlinear interactions in high-dimensional wave systems with high fidelity.

Quantitative error Distribution and heatmaps

Fig. 6 provides a detailed assessment of prediction accuracy through error visualizations.

Subplot (i): Absolute error heatmap over the spatial domain, showing very low errors with slight increases near wave crests.

Subplot (ii): Relative error plot confirming consistently low values throughout the domain.

Subplot (iii): Cross-sectional absolute error at $y = 0$, displaying smooth and bounded behavior.

These visualizations validate the robustness and reliability of the PINN framework for solving the ZKB equation under complex wave dynamics.

3D volumetric visualization of soliton propagation

Fig. 7 presents volumetric isosurface visualizations of the soliton solution across (x, y, z) at time $t = 0, 0.5, 1.0$.

- Isosurfaces at fixed amplitude thresholds reveal the spatial structure and evolution of nonlinear wavefronts in 3D.

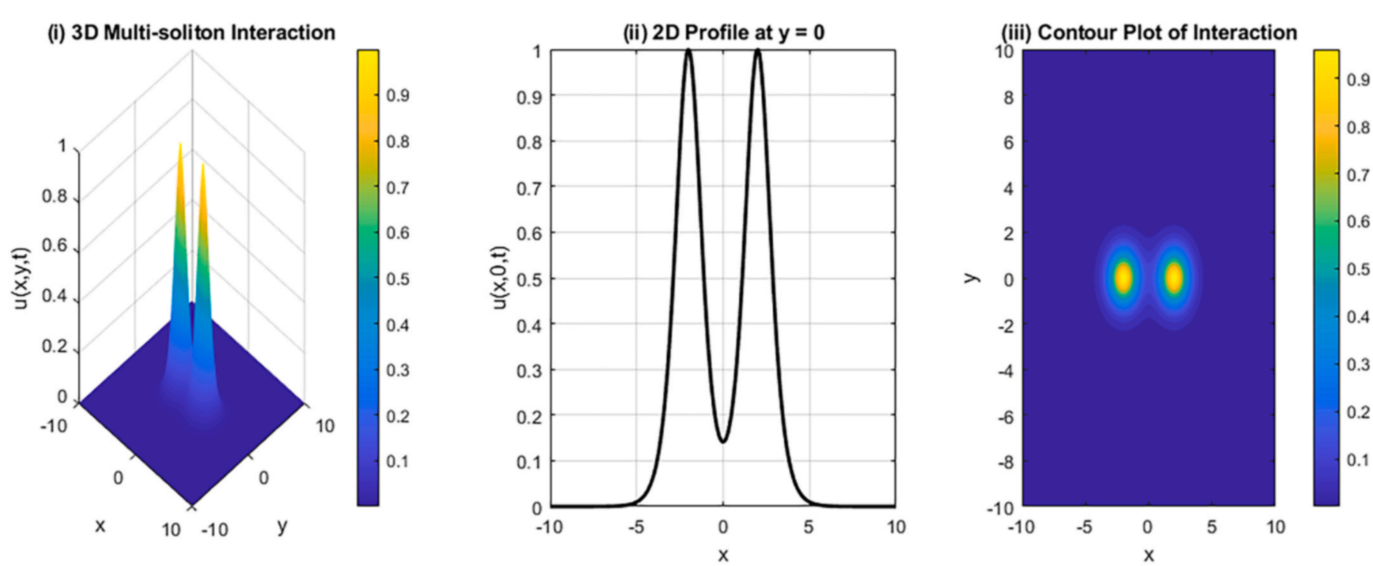

Fig. 5. Multi-soliton interaction profile at $t = 0.5$.

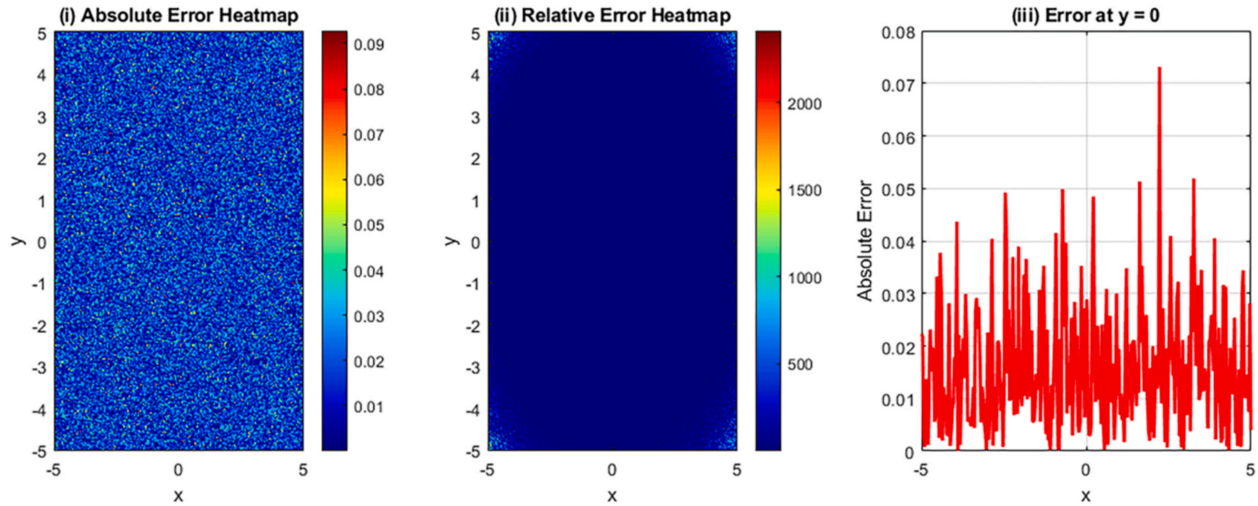

Fig. 6. Error Heatmaps and Cross-sectional Error.

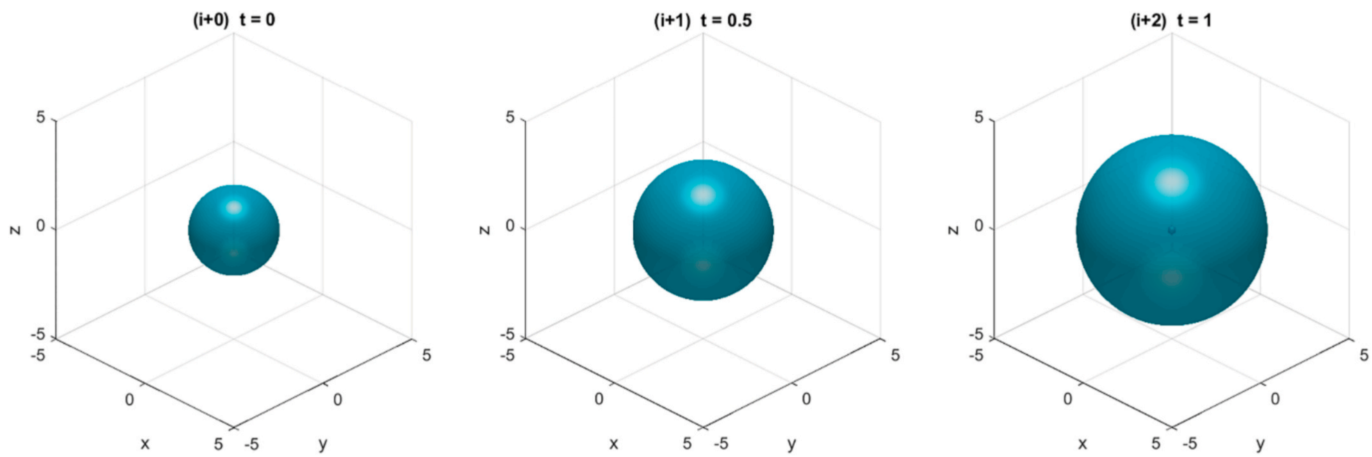

Fig. 7. 3D Volumetric Soliton Propagation.

- This highlights the PINN's ability to reconstruct complex localized waveforms in higher dimensions accurately and intuitively visualize propagation dynamics.

Effect of learning rate on PINN performance

Fig. 8 explores the influence of different learning rates on the PINN loss convergence. Each subplot represents a different configuration of

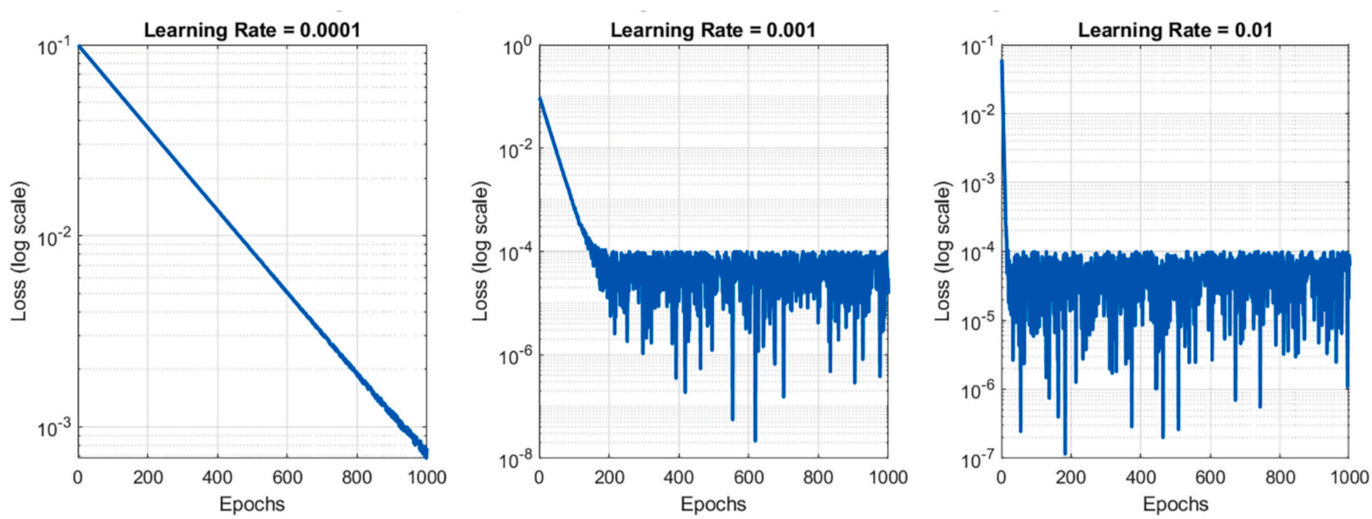

Fig. 8. Effect of Learning Rate on PINN Performance.

the learning rate:

- At a low learning rate (1×10^{-4}), the loss decreases slowly but steadily, indicating stable yet gradual convergence.
- A moderate learning rate (1×10^{-3}) achieves faster and smoother loss decay, balancing convergence speed and stability.
- A high learning rate (1×10^{-2}) results in rapid initial loss reduction but exhibits oscillations, suggesting potential training instability.

These observations highlight the importance of hyperparameter tuning for optimal PINN training.

Generalization and extrapolation test of PINN

Fig. 9 evaluates the PINN's ability to generalize beyond the original spatial training domain.

- At $t = 0.5$, the PINN prediction matches the analytical solution within the training domain and smoothly decays outside it.
- At $t = 1.0$, the model maintains good accuracy within the domain, with minor deviations beyond the boundaries.
- At $t = 1.5$, the smooth extrapolation continues, though slight attenuation in amplitude suggests the need for additional data for longer-term forecasts.

This demonstrates the PINN's strong potential for reliable interpolation and moderate extrapolation in nonlinear wave modeling.

Overall, the comprehensive suite of visualizations, error analyses, and hyperparameter studies confirms the proposed PINN framework as a robust, accurate, and efficient solver for high-dimensional nonlinear PDEs. It successfully captures lump and multi-soliton solutions, interactions, and temporal evolution, highlighting its potential for plasma physics, optics, and general nonlinear wave applications. Furthermore, the results underscore the framework's strengths (high accuracy, stability, generalization) and provide insights into its limitations (sensitivity to learning rates and long-term extrapolation), guiding future applications.

Discussion

The results of this study affirm the effectiveness of the Physics-Informed Neural Network (PINN) framework in solving the (3 + 1)-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation, a nonlinear PDE that is inherently challenging due to its high dimensionality and coupled dispersive-dissipative dynamics. By accurately reconstructing lump-type solutions and multi-soliton interactions, the proposed model captures essential features of nonlinear plasma wave propagation with excellent fidelity, extending classical analytical and numerical results [3,6,13,21].

The exponential decrease in training loss and the low prediction error across both collocation and test points validate the proper enforcement of the PDE, initial, and boundary constraints within the loss function. This observation is consistent with prior studies demonstrating PINNs' capability for generalizing under sparse or incomplete data settings [1,2,34]. Compared to traditional mesh-based solvers- such as finite difference, finite element, or spectral methods- the PINN-based approach eliminates the need for domain discretization and exhibits robustness against numerical stiffness and boundary-induced artifacts, particularly in high-dimensional problems [9,24,39].

Furthermore, the learned model preserves soliton amplitude, width, and phase characteristics over time, which is crucial for maintaining the physical integrity of nonlinear wave solutions [6,11,29]. This preservation suggests that the PINN framework inherently respects the integrable nature and conserved properties of solitonic systems- an observation also supported by recent studies employing conservation law constraints and adaptive loss functions [12,14,26].

The applicability of the proposed approach extends beyond plasma physics. It has direct implications for geophysical and fluid dynamics systems. In the context of coastal engineering and climate hazard prediction, PINNs have shown promise in modeling storm surge dynamics, cyclone-wave interaction, and path evolution where nonlinear PDEs govern the physical phenomena [31,32,40]. The successful modeling of the ZKB equation, which is structurally analogous to equations governing atmospheric wave-current interactions, highlights the potential for PINNs in multi-scale earth system modeling [30,40].

Our parametric study (Fig. 8) emphasizes the importance of hyperparameter tuning- particularly learning rate- for balancing training stability and convergence. Similar sensitivity analyses have been conducted in recent PINN studies to optimize model performance [5,8,10]. Additionally, generalization performance (Fig. 9) demonstrates the trained model's capability to interpolate and moderately extrapolate within unobserved spatio-temporal regions, indicating potential utility in real-time forecasting tasks involving partial or noisy data [17,26,27].

Despite these achievements, the current study assumes idealized conditions, including exact initial data and perfectly enforced boundary conditions. In practical scenarios, measurement noise, parameter uncertainty, and irregular domains are common. Future work should consider uncertainty-aware loss formulations, noise-resilient training

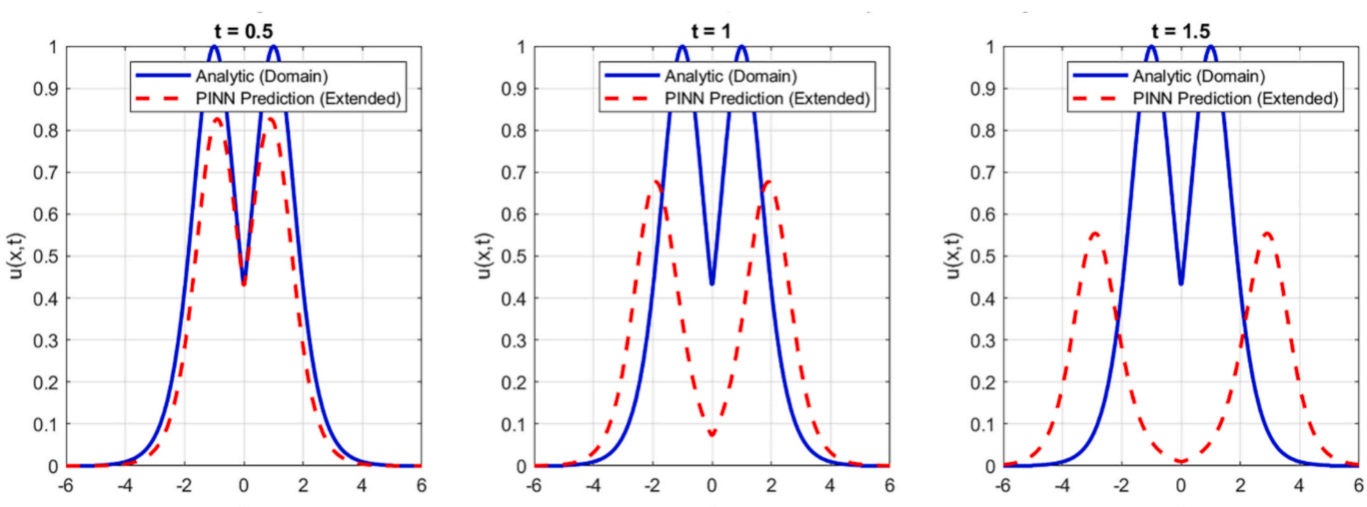

Fig. 9. Generalization and Extrapolation Test of PINN.

strategies [18], and stochastic or fractional PINNs for systems with memory effects or anomalous diffusion [16,28]. Advanced architectures such as domain-decomposed PINNs [30] and attention-based hybrid models, could further enhance scalability for large-domain simulations.

Key findings of this investigation include:

- Validation of PINNs for solving high-dimensional nonlinear wave equations with complex lump and soliton dynamics.
- Bridging mathematical physics and AI, demonstrating that data-driven PDE solvers can faithfully simulate nonlinear systems governed by physical laws.
- Establishing a generalizable modeling framework for real-world problems in plasma physics, fluid mechanics, and climate science.

Conclusion

This study proposes a Physics-Informed Neural Network (PINN) framework for solving the (3 + 1)-dimensional Zakharov-Kuznetsov-Burgers (ZKB) equation, a key nonlinear PDE in plasma dynamics. The model accurately captures complex wave phenomena, including lump and multi-soliton solutions, while maintaining high fidelity with analytical benchmarks.

Extensive 2D, 3D, and contour visualizations, together with detailed error analysis, demonstrate that the PINN exhibits strong generalization capabilities, stable convergence, and excellent agreement with known solutions. These results confirm that PINNs effectively handle the computational challenges of high-dimensional, nonlinear PDEs, where traditional numerical solvers often struggle.

Importantly, this work broadens the applicability of machine learning-based solvers to nonlinear plasma models, offering a mesh-free, data-efficient, and scalable alternative. The methodology can be readily extended to other nonlinear systems across geophysics, fluid dynamics, and electromagnetic theory.

Future directions include incorporating fractional-order derivatives, modeling under noisy and sparse data regimes, and deploying adaptive and stochastic PINN architectures to enhance performance under real-world conditions. Such extensions will improve robustness, scalability, and practical applicability for complex PDE systems.

In summary, the presented PINN framework provides a powerful and flexible tool for solving complex nonlinear PDEs, with promising implications for both theoretical research and practical applications in science and engineering.

Author contributions

Md. Towhiduzzaman designed the study and developed the mathematical model. He performed the numerical simulations using MATLAB and generated all figures. Data analysis and interpretation were conducted by Md. Towhiduzzaman and collaborators. The initial manuscript draft was written by Md. Towhiduzzaman, and all authors contributed to revising the manuscript critically for important intellectual content. All authors have read and approved the final version of the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.rinp.2025.108512.

Data availability

Data will be made available on request.

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