Dynamical structures of exact soliton solutions to Burgers' equation via the bilinear approach

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Dynamical structures of exact soliton solutions to Burgers' equation via the bilinear approach

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ABSTRACT

The Cole–Hopf transformation is used to search for exact multiple wave solutions of Burgers' equation. Particularly, solitary wave solutions and their interaction solutions are explicitly given, and a fission phenomenon is found, which is tough to be found by other traditional methods. Different solutions are presented with different combinations of elementary functions and a necessary condition for fission is proposed. Based on the bilinear formalism and with the aid of symbolic computation, we also determine two classes of rational solutions, rogue wave solutions and rogue–kink wave solutions, using various ansätze. To exhibit dynamics, we make diverse graphical analyses for the presented solutions, which show that the solutions are reliable in both mathematical physics and engineering.

1. Introduction

Soliton equations connect affluent histories of exactly solvable systems formed in mathematics, microphysics, fluid physics, cosmology, field theory, etc. To explain some physical phenomenon further, it becomes more and more important to seek exact solutions and interactions among nonlinear wave solutions. Owing to the wide applications of soliton theory in various field of natural science such as plasma physics, mathematics, biology, nuclear physics, hydrodynamics, astrophysics and geophysics, the study on integrable models has attracted much attention of many researchers. With the development of soliton theory, many kinds of powerful methods were proposed to finding the exact solutions of nonlinear partial differential equations like inverse scattering transformation,[1] (G'/G)-expansion method,[2] exp-function method,[3–5] Hirota's direct method,[6–16] Rational function method,[17] the sine–cosine method[18] etc. Yet some soliton models gives completely non-elastic soliton interactions when specific conditions between the wave vectors and velocities are satisfied; at a specific time, one soliton may fission to two or more solitons (soliton fission phenomena) or two or more solitons will fusion to one soliton (soliton fusion phenomena). Actually, for numerous genuine physical models such as in macromolecule material and organic membrane,[19] in even-clump DNA[20] and in many physical fields like plasma physics, nuclear physics and so on Refs. 21–25, people have observed the same phenomena. Wazwaz[26–28] investigated multiple soliton solutions such type of non-elastic phenomena. Burgers' equation and Sharma–Tasso–Olver equation are such types of model in which Wang et al.[29] found non-elastic soliton fission and fusion phenomena with only two dispersion relations. Alam et al.,[30] obtain some hyperbolic, trigonometric and rational function solutions of Burgers' equation via (G'/G)-expansion method.

The main purpose of this paper is to investigate non-elastic fission phenomenon to (1+1) - dimensional Burgers' equation using the bilinear approach and a direct test function. And obtain new solitary wave and rogue wave solutions which are addressed with some figures.

This paper is organized as. In Sections 2 and 3 two types of solutions such as multiple wave (single wave, two-soliton and three-soliton) solutions and rational polynomial function solutions of the Burgers' equation are illustrated. In Sections 4 and 5, rogue wave solution and its interaction with solitary wave are demonstrated. Finally, the conclusions are presented in Section 6.

2. Multiple wave solutions of Burgers' equation and their fission

The (1+1)-dimensional Burgers' equation[29,30] reads as

$$u_t + H u_{xx} + u u_x = 0, \tag{2.1}$$

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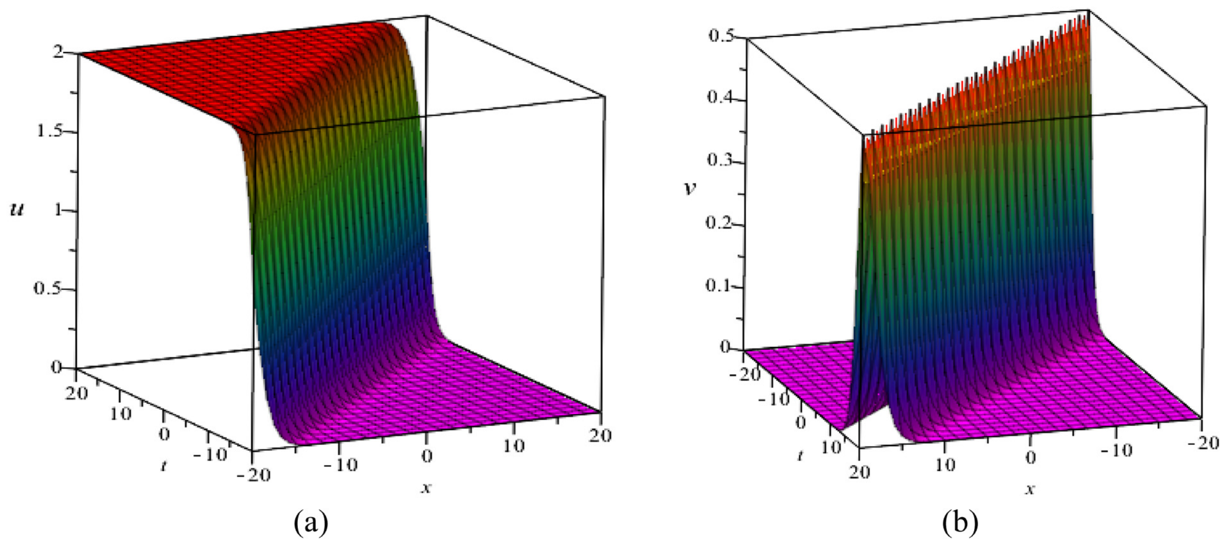

Fig. 1. Profile of single solitary wave solution for Eq. (2.1) by choosing suitable parameters: $H = k_1 = -1, a_0 = c_1 = 1$ (a) Perspective view of the wave Eq. (2.5) (b) Perspective view of the wave Eq. (2.6).

where H is a constant to be determined later. Eq. (2.1) is a fundamental partial differential equation occurring in various areas of applied mathematics, such as fluid mechanics, nonlinear acoustics, gas dynamics, and traffic flow.

Under the dependent variable transformation

$$u = 2H(\ln f)_x. \tag{2.2}$$

Eq. (2.1) becomes the following bilinear form

$$ff_{xt} - f_x f_t + H\left(ff_{xxx} - f_x f_{xx}\right) = 0. \tag{2.3}$$

For single soliton solution of Eq. (2.1), we first consider

$$f(x,t) = a_0 + c_1 \exp\left(k_1 x + w_1 t\right), \tag{2.4}$$

and the corresponding potential field reads $v = -u_x$.

Inserting Eq. (2.4) with Eq. (2.2) into bilinear form Eq. (2.3), we can find the value of a_0, w_1 as $a_0 = const., w_1 = -Hk_1^2$. If we substitute the value of a_0, w_1 into Eq. (2.3) along with Eq. (2.4) then we obtain following single soliton solution:

$$u(x,t) = \frac{2Hk_1c_1 \exp\left(k_1\left(x - Hk_1t\right)\right)}{a_0 + c_1 \exp\left(k_1\left(x - Hk_1t\right)\right)}, \tag{2.5}$$

and corresponding potential field reads as

$$v(x,t) = \frac{2Hk_1^2c_1^2 \exp\left(2k_1\left(x - Hk_1t\right)\right)}{\left(a_0 + c_1 \exp\left(k_1\left(x - Hk_1t\right)\right)\right)^2} - \frac{2Hk_1^2c_1 \exp\left(k_1\left(x - Hk_1t\right)\right)}{a_0 + c_1 \exp\left(k_1\left(x - Hk_1t\right)\right)}. \tag{2.6}$$

In what follows, Fig. 1 presents single solitary wave solution and its potential field of Eq. (2.5) by choosing the suitable parameters.

To achieve two soliton solutions of Eq. (2.1), we just suppose

$$f(x,t) = a_0 + c_1 \exp\left(\xi_1\right) + c_2 \exp\left(\xi_2\right) + a_{12}c_1c_2 \exp\left(\xi_1 + \xi_2\right), \tag{2.7}$$

where $\xi_1 = k_1x + w_1t, \xi_2 = k_2x + w_2t$ and the corresponding potential field reads $v = -u_x$.

Setting Eq. (2.7) with Eq. (2.2) into the bilinear form Eq. (2.3), we gain some polynomials which are functions of the variables x and t. Equating all the coefficients of different power of exponential to zero, we can obtain a system of algebraic equations in terms of $k_1, k_2, c_1, c_2, w_1, w_2$ and a_{12}. Solving the system of algebraic equations with the help of symbolic computation system Maple, we get some solution sets of the unknown parameters which are given below.

Set-1: $a_0 = const., a_{12} = 0, w_1 = -Hk_1^2, w_2 = -Hk_2^2$, then

$$u(x,t) = 2H\left(\frac{k_1c_1 \exp\left(k_1\left(x - Hk_1t\right)\right) + k_2c_2 \exp\left(k_2\left(x - Hk_2t\right)\right)}{a_0 + c_1 \exp\left(k_1\left(x - Hk_1t\right)\right) + c_2 \exp\left(k_2\left(x - Hk_2t\right)\right)}\right), \tag{2.8}$$

where $a_0, c_1, c_2, k_1, k_2, H$ are arbitrary constants.

The corresponding potential field read as

$$v(x,t) = 2H\left(\frac{k_1c_1 \exp\left(\eta_1\right) + k_2c_2 \exp\left(\eta_2\right)}{a_0 + c_1 \exp\left(\eta_1\right) + c_2 \exp\left(\eta_2\right)}\right)^2 - 2H\left(\frac{k_1^2c_1 \exp\left(\eta_1\right) + k_2^2c_2 \exp\left(\eta_2\right)}{a_0 + c_1 \exp\left(\eta_1\right) + c_2 \exp\left(\eta_2\right)}\right), \tag{2.9}$$

with $\eta_1 = k_1\left(x - Hk_1t\right)$, and $\eta_2 = k_2\left(x - Hk_2t\right)$.

From Fig. 2, the plots of the fission phenomenon between two solitary waves with the parameters choosing as $k_1 = -1, k_2 = -1.5, H = a_0 = c_1 = c_2 = 1$, we can evidently see that, after interaction one single solitary wave fission to two solitary wave and decrease wave height with smaller potential energy. i.e., at a specific time $t = 0$. That is, when the two populations living nearer to each other after interaction between them with different characteristics and tend to be a single clannish, which is completely different from the previous nature. From careful analysis of Eq. (2.8), Eq. (2.9), it concludes that, only fission phenomena occurs for all the ranges of two arbitrary parameters k_1, k_2. Neither elastic scattering nor fusion does exist.

Set-2: $a_0 = 0, a_{12} = const., w_1 = -H\left(k_1^2 + 2k_1k_2\right)$, $w_2 = -H\left(k_2^2 + 2k_1k\right)$, then

$$u(x,t) = 2H\left(\frac{k_1c_1 \exp\left(\xi_1\right) + k_2c_2 \exp\left(\xi_2\right) + a_{12}\left(k_1 + k_2\right)c_1c_2 \exp\left(\xi_1 + \xi_2\right)}{c_1 \exp\left(\xi_1\right) + c_2 \exp\left(\xi_2\right) + a_{12}c_1c_2 \exp\left(\xi_1 + \xi_2\right)}\right), \tag{2.10}$$

where $\xi_1 = k_1x - H\left(k_1^2 + 2k_1k_2\right)t, \xi_2 = k_2x - H\left(k_2^2 + 2k_1k\right)t$ and c_1, c_2, k_1, k_2, H are arbitrary constants and the corresponding potential field reads $v = -u_x$. This solution also has the same phenomena like solution Eq. (2.10).

Set-3: $a_0 = 0, a_{12} = 0, w_1 = H\left(k_2^2 - k_1^2\right) + w_2, w_2 = const.$, then

$u(x,t)$

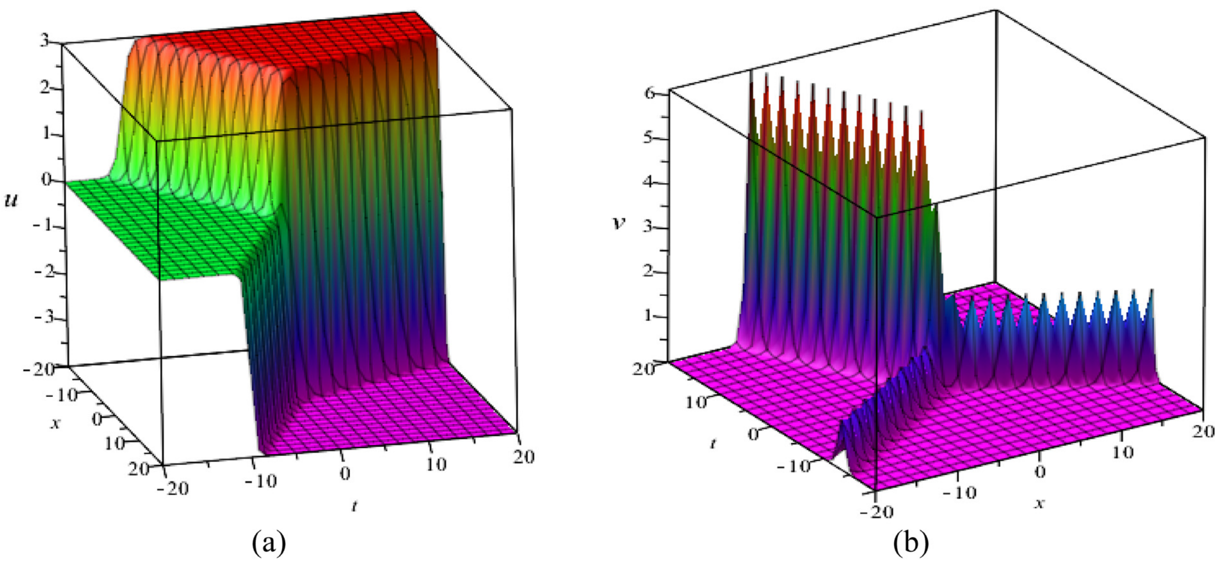

Fig. 2. Profile of two solitary wave solution for Eq. (2.1) by choosing suitable parameters: $H=-1, k_1=2, k_2=-1.5$, $a_0=c_1=c_2=1$ (a) Perspective view of the wave Eq. (2.8) (b) Perspective view of the wave Eq. (2.9).

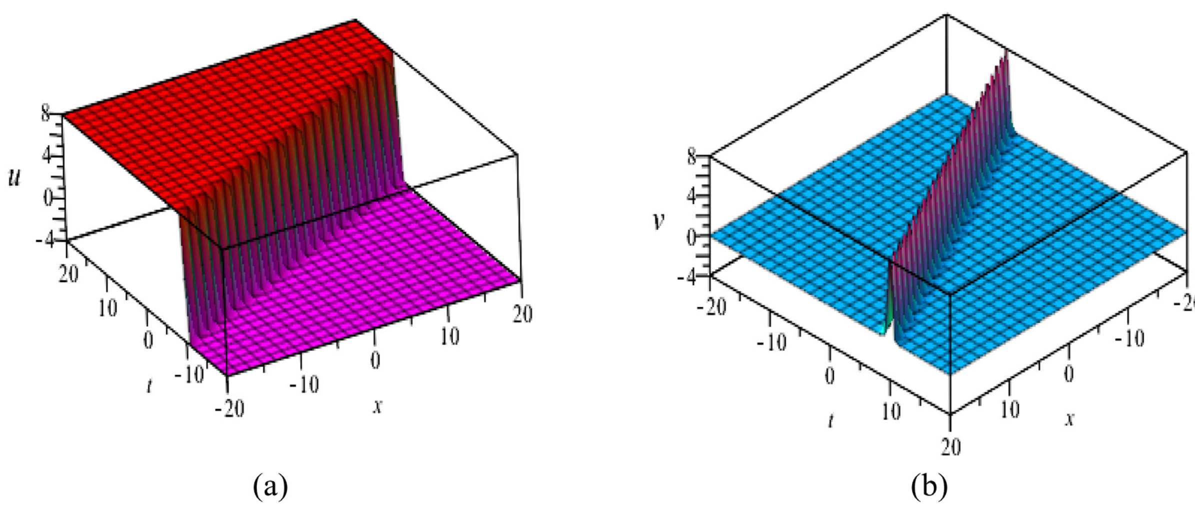

Fig. 3. Profile of two solitary wave solution for Eq. (2.1) by choosing suitable parameters: $H=k_2=-2, w_2=0.5$, $k_1=c_1=c_2=1$ (a) Perspective view of the wave Eq. (2.11) (b) Perspective view of corresponding potential field.

$$= 2H\left(\frac{k_1c_1\exp\left(k_1x+\left(H\left(k_2^2-k_1^2\right)+w_2\right)t\right)+k_2c_2\exp\left(k_2x+w_2t\right)}{c_1\exp\left(k_1x+\left(H\left(k_2^2-k_1^2\right)+w_2\right)t\right)+c_2\exp\left(k_2x+w_2t\right)}\right), \tag{2.11}$$

where c_1, c_2, k_1, k_2, H are arbitrary constants.

The corresponding potential field reads $v=-u_x$.

From Fig. 3 solution Eq. (2.11) is completely elastic. That is to say, the soliton amplitude, wave shape and velocity remain same after non-linear collisions. So, elastic scattering, fusion and fission phenomena do not exist for any values of the parameters.

To achieve three soliton solutions of Eq. (2.1), we just suppose

$$\begin{aligned} f(x,t) &= a_0+c_1\exp\left(\xi_1\right)+c_2\exp\left(\xi_2\right)+c_3\exp\left(\xi_3\right)\\ &+a_{12}c_1c_2\exp\left(\xi_1+\xi_2\right)+\\ &a_{23}c_2c_3\exp\left(\xi_2+\xi_3\right)+a_{13}c_1c_3\exp\left(\xi_1+\xi_3\right)\\ &+a_{123}c_1c_2c_3\exp\left(\xi_1+\xi_2+\xi_3\right) \end{aligned}, \tag{2.12}$$

where $\xi_1=k_1x+w_1t, \xi_2=k_2x+w_2t, \xi_3=k_3x+w_3t$ and the corresponding potential field reads $v=-u_x$.

Setting Eq. (2.12) with Eq. (2.2) into the bilinear form Eq. (2.3), we gain some polynomials which are functions of the variables x and t. Equating all the coefficients of different power of exponential to zero, we can obtain a system of algebraic equations in terms of $k_1, k_2, k_3, w_1, w_2, w_3, c_1, c_2, c_3, a_{12}, a_{23}, a_{13}$ and a_{123}. Solving the system of algebraic equations with the aid of symbolic computation system Maple, we attain the following solutions of the unknown parameters.

Set-1: $a_0=const., a_{12}=a_{23}=a_{13}=a_{123}=0, w_1=-Hk_1^2, w_2=-Hk_2^2, w_3=-Hk_3^2$, then

$$u(x,t) = 2H\left(\frac{k_1c_1\exp\left(k_1\left(x-Hk_1t\right)\right)+k_2c_2\exp\left(k_2\left(x-Hk_2t\right)\right)+k_3c_3\exp\left(k_3\left(x-Hk_3t\right)\right)}{a_0+c_1\exp\left(k_1\left(x-Hk_1t\right)\right)+c_2\exp\left(k_2\left(x-Hk_2t\right)\right)+c_3\exp\left(k_3\left(x-Hk_3t\right)\right)}\right), \tag{2.13}$$

where $a_0, k_1, k_2, k_3, c_1, c_2, c_3, H$ are arbitrary constants.

The corresponding potential field reads $v=-u_x$.

In Fig. 4, the plots of the fusion phenomenon of the three solitons (shock waves) with the parameters choosing as: $H=a_0=k_1=c_1=c_2=c_3=1, k_2=-1.5, k_3=2$, it is clearly shown that three single soliton fusions present one (resonant) soliton after the interaction, i.e., for a specific time $t=0$.

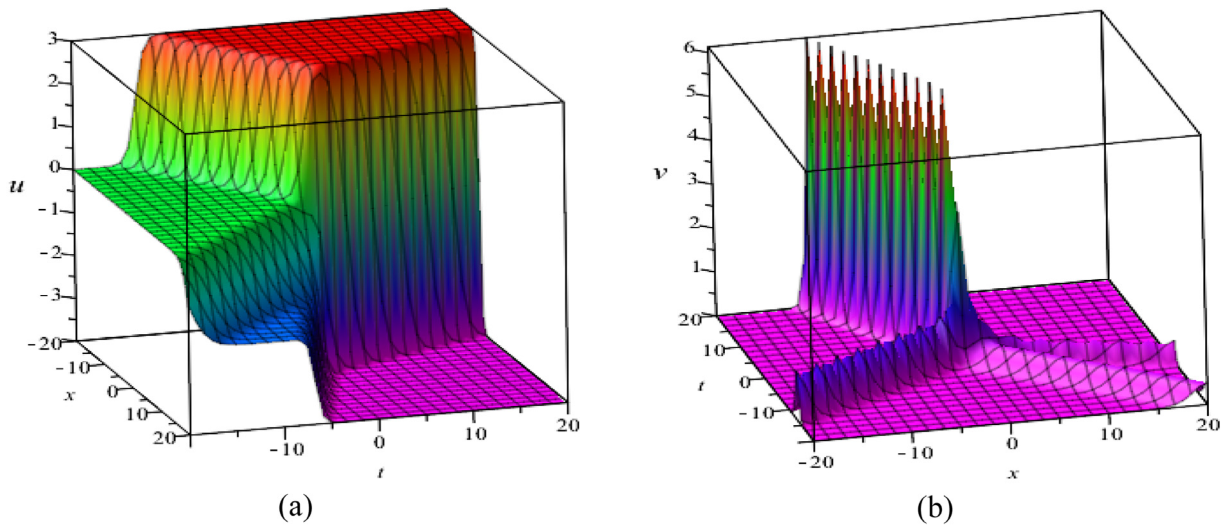

Fig. 4. Profile of three solitary wave fusion solution for Eq. (2.1) by choosing suitable parameters: $H=-1, a_0=k_1=c_1=c_2=c_3=1, k_2=-1.5, k_3=2$ (a) Perspective view of the wave Eq. (2.13) (b) Perspective view of corresponding potential field.

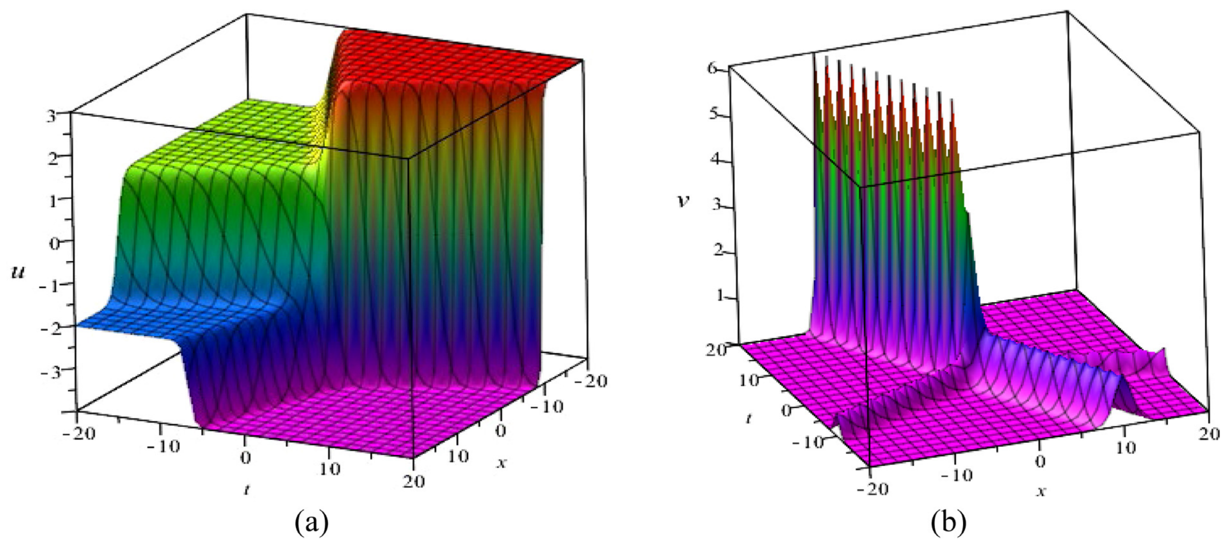

Fig. 5. Profile of three solitary wave fusion solution for Eq. (2.1) by choosing suitable parameters: $H=-1, a_{12}=k_1=c_1=c_2=c_3=1, k_2=-1.5, k_3=2$ (a) Perspective view of the wave Eq. (2.14) (b) Perspective view of corresponding potential field.

Set-2: $a_{12}=const., a_0=a_{23}=a_{13}=a_{123}=0, w_1=-H\left(k_1^2+2k_1k_2\right)$, $w_2=-H\left(k_2^2+2k_1k_2\right)$, $w_3=-H\left(k_3^2+2k_1k_2\right)$, then

$$u(x,t) = 2H\left(\frac{k_1c_1\exp(\xi_1)+k_2c_2\exp(\xi_2)+k_3c_3\exp(\xi_3)+a_{12}(k_1+k_2)c_1c_2\exp(\xi_1+\xi_2)}{c_1\exp(\xi_1)+c_2\exp(\xi_2)+c_3\exp(\xi_3)+a_{12}c_1c_2\exp(\xi_1+\xi_2)}\right), \tag{2.14}$$

where $\xi_1=k_1x-H\left(k_1^2+2k_1k_2\right)t, \xi_2=k_2x-H\left(k_2^2+2k_1k_2\right)t, \xi_3=k_3x-H\left(k_3^2+2k_1k_2\right)t$, and the corresponding potential field reads $v=-u_x$ (see Fig. 5).

Set-3: $a_{23}=const., a_0=a_{12}=a_{13}=a_{123}=0, w_1=-H\left(k_1^2+2k_2k_3\right)$, $w_2=-H\left(k_2^2+2k_2k_3\right)$, $w_3=-H\left(k_3^2+2k_2k_3\right)$, then

$$u(x,t) = 2H\left(\frac{k_1c_1\exp(\xi_1)+k_2c_2\exp(\xi_2)+k_3c_3\exp(\xi_3)+a_{23}(k_2+k_3)c_2c_3\exp(\xi_2+\xi_3)}{c_1\exp(\xi_1)+c_2\exp(\xi_2)+c_3\exp(\xi_3)+a_{23}c_2c_3\exp(\xi_2+\xi_3)}\right), \tag{2.15}$$

where $\xi_1=k_1x-H\left(k_1^2+2k_2k_3\right)t, \xi_2=k_2x-H\left(k_2^2+2k_2k_3\right)t, \xi_3=k_3x-H\left(k_3^2+2k_2k_3\right)t$, and the corresponding potential field reads $v=-u_x$.

Set-4: $a_{13}=const., a_0=a_{12}=a_{23}=a_{123}=0, w_1=-H\left(k_1^2+2k_1k_3\right)$, $w_2=-H\left(k_2^2+2k_1k_3\right)$, $w_3=-H\left(k_3^2+2k_1k_3\right)$, then

$$u(x,t) = 2H\left(\frac{k_1c_1\exp(\xi_1)+k_2c_2\exp(\xi_2)+k_3c_3\exp(\xi_3)+a_{13}(k_1+k_3)c_1c_3\exp(\xi_1+\xi_3)}{c_1\exp(\xi_1)+c_2\exp(\xi_2)+c_3\exp(\xi_3)+a_{13}c_1c_3\exp(\xi_1+\xi_3)}\right), \tag{2.16}$$

where $\xi_1=k_1x-H\left(k_1^2+2k_1k_3\right)t, \xi_2=k_2x-H\left(k_2^2+2k_1k_3\right)t, \xi_3=k_3x-H\left(k_3^2+2k_1k_3\right)t$, and the corresponding potential field reads $v=-u_x$.

Set-5: $a_{123}=const., a_0=a_{12}=a_{23}=a_{13}=0$, $w_1=-H\left(k_1^2+k_1k_2+k_1k_3+k_2k_3\right)$, $w_2=-H(k_2^2+k_1k_2+k_1k_3+k_2k_3)$, $w_3=-H\left(k_3^2+k_1k_2+k_1k_3+k_2k_3\right)$, then

$$u(x,t) = 2H\left(\frac{k_1c_1\exp(\xi_1)+k_2c_2\exp(\xi_2)+k_3c_3\exp(\xi_3)+a_{123}\Delta c_1c_2c_3\exp(\xi_1+\xi_2+\xi_3)}{c_1\exp(\xi_1)+c_2\exp(\xi_2)+c_3\exp(\xi_3)+a_{123}c_1c_2c_3\exp(\xi_1+\xi_2+\xi_3)}\right), \tag{2.17}$$

where $\Delta=\left(k_1+k_2+k_3\right)$, $\xi_1=k_1x-H\left(k_1^2+k_1k_2+k_1k_3+k_2k_3\right)t, \xi_2=k_2x-H(k_2^2+k_1k_2+k_1k_3+k_2k_3)t$, $\xi_3=k_3x-H\left(k_3^2+k_1k_2+k_1k_3+k_2k_3\right)t$, and the corresponding potential field reads $v=-u_x$.

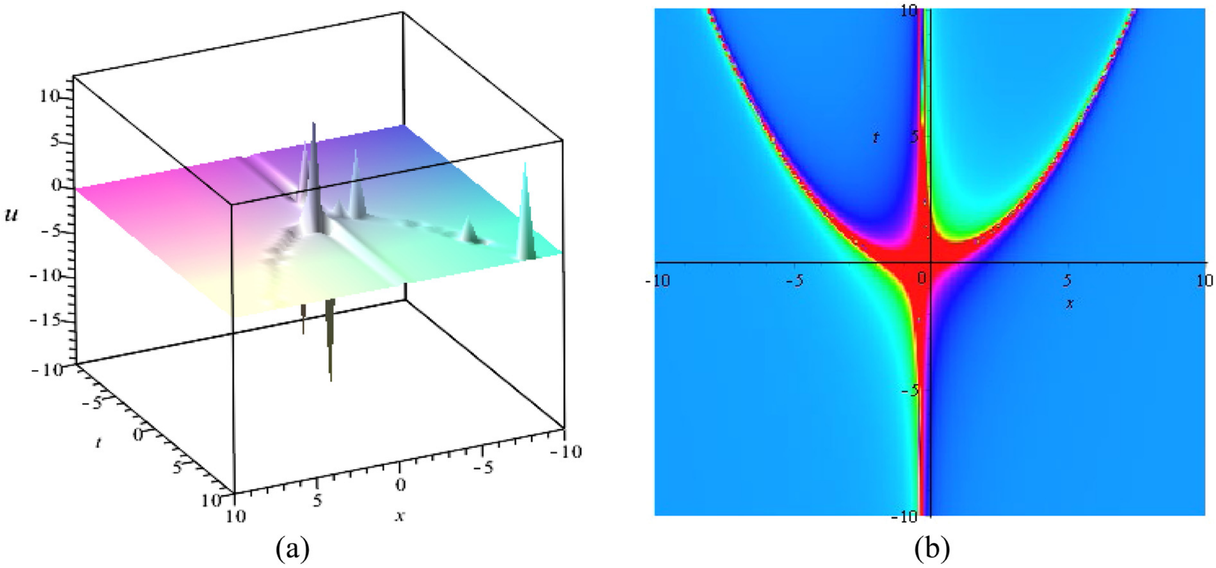

Fig. 6. Profile of rational polynomial solution for Eq. (2.1) by choosing suitable parameters: $H = c = 1$, $a = b = 2, d = 3$ (a) Perspective view of the wave Eq. (3.3) (b) corresponding density plot.

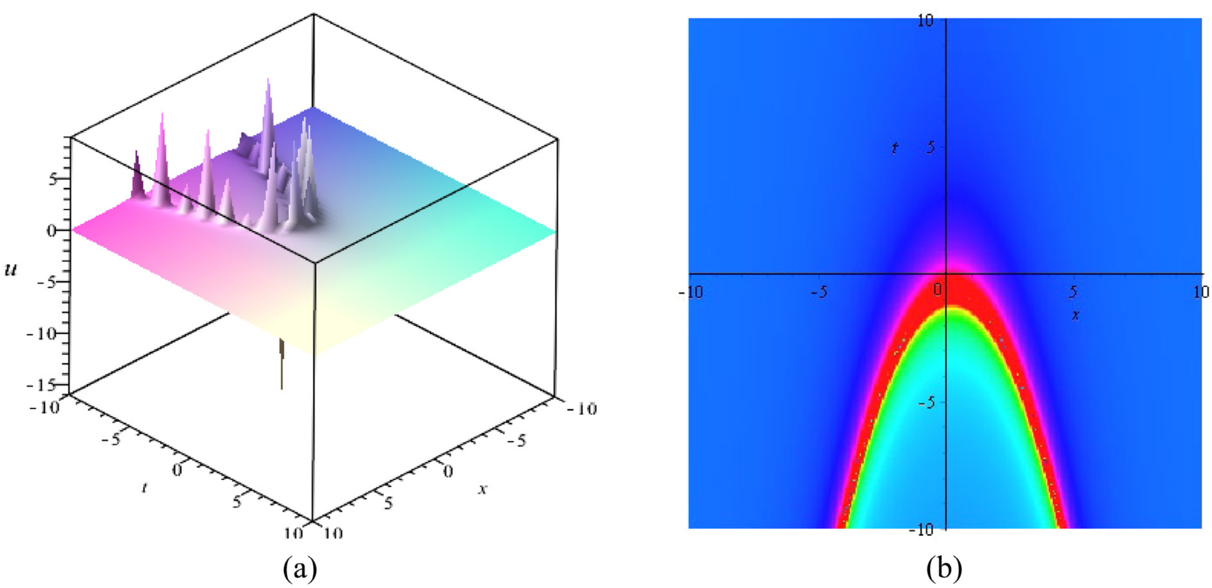

Fig. 7. Profile of rational polynomial solution for Eq. (2.1) by choosing suitable parameters: $H = 1, a = 2$, $b = -1, c = 3$ (a) Perspective view of the wave Eq. (3.4) (b) corresponding density plot.

Set-6: $a_{123} = a_0 = a_{12} = a_{13} = a_{23} = 0, w_1 = const.$, $w_2 = H\left(k_1^2 - k_2^2\right) + w_1, w_3 = H\left(k_1^2 - k_3^2\right) + w_1$, then

$$u(x,t) = 2H\left(\frac{k_1 c_1 \exp\left(\xi_1\right) + k_2 c_2 \exp\left(\xi_2\right) + k_3 c_3 \exp\left(\xi_3\right)}{c_1 \exp\left(\xi_1\right) + c_2 \exp\left(\xi_2\right) + c_3 \exp\left(\xi_3\right)}\right), \tag{2.18}$$

where $\xi_1 = k_1 x - w_1 t, \xi_2 = k_2 x + \left(H\left(k_1^2 - k_2^2\right) + w_1\right) t, \xi_3 = k_3 x + \left(H\left(k_1^2 - k_3^2\right) + w_1\right)$, k_1, k_2, k_3, c_1, c_2, c_3 are arbitrary constants and the corresponding potential field reads $v = -u_x$.

3. Rational function solutions of Burgers' equation

By a Maple computation on

$$f = \sum_{i=0}^{7}\sum_{j=0}^{7} C_{ij} x^i t^j \tag{3.1}$$

we notice that such type of polynomial solution for Eq. (2.3) does not allow the degree of t greater than 1 which is analytically proved by Ma.[31] He made a hypothesis that this is true, namely, any polynomial solution f to the bilinear equation (2.3) must have the degree of t not greater than 1. In this regard, we want to consider the trial solution for the rational polynomial is of degree as follows:

$$f = R(x)t + w(x), \tag{3.2}$$

with $R(x)$ of degree one and $w(x)$ of degree at least three.

According to the bilinear form Eq. (2.3), we have to write $R(x) = apx + qb$, $w(x) = ax^3 + bx^2 + cx + d$.

Thus using Eq. (3.2) into the bilinear equation like Eq. (2.3) and solving, we achieve a class of rational polynomial solutions of the Eq. (2.1):

$$u(x,t) = \frac{2H\left(-6Hat + 3ax^2 + 2bx + c\right)}{-(6Hax + 2bH)t + ax^3 + bx^2 + cx + d}. \tag{3.3}$$

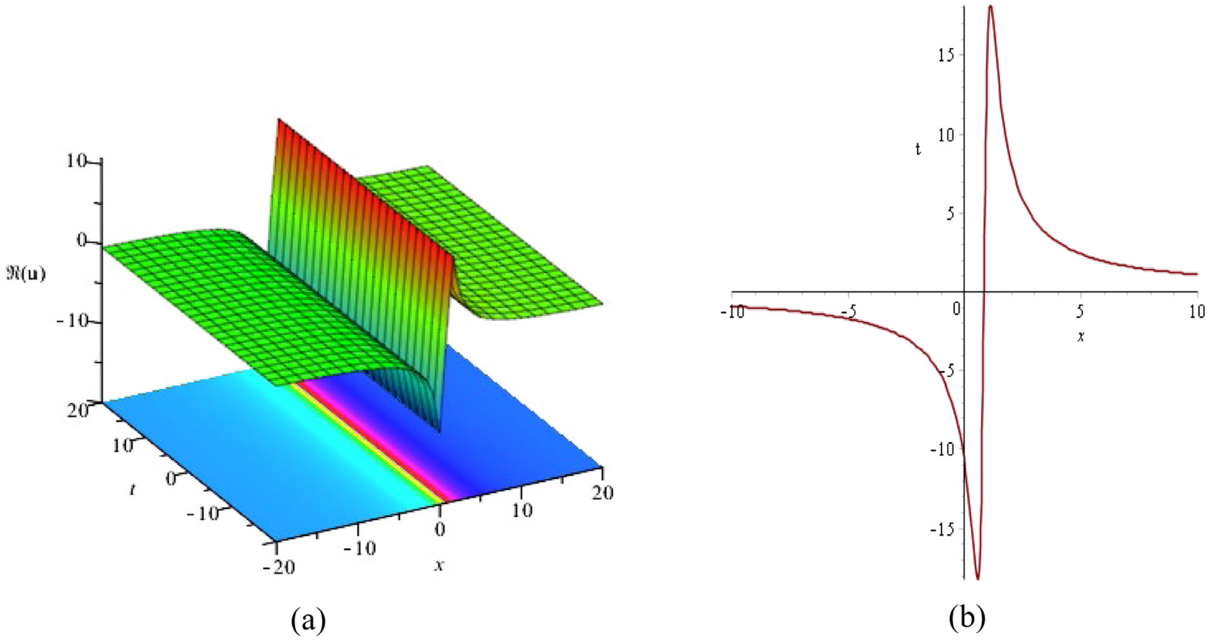

Fig. 8. Profile of Line Rogue wave (4.5) for Eq. (2.1) by choosing suitable parameters: $H = 5$, $a_1 = 2\sqrt{-1}$,$a_3 = 1$ and $a_6 = 3$.(a) Perspective view of the wave and corresponding density plot (bottom) (b) The wave propagation pattern of the wave along the x axis.

On the other hand, if we consider $R(x)$ of degree two and $w(x)$ of degree four, then setting $f = R(x)t + w(x)$ with $R(x) = pax^2qbx + lc$, $w(x) = ax^4 + bx^3 + cx^2 + dx + k$ into the bilinear form Eq. (2.3) and solving for coefficients of different powers of $x^i t^j$ for a, p, q, and l, we get $a = 0, p = const., q = -6H, l = -2H$, which are the same results of Eq. (3.3). Now, we can conclude that in the rational polynomial solutions $w(x)$ does not support as a polynomial of degree more than 3 (see Fig. 6).

Secondly, we consider $R(x)$ of degree two and $w(x)$ of degree two, then setting $f = R(x)t + w(x)$ with $R(x) = apx + qb$, $w(x) = ax^2 + bx + c$ into the bilinear form Eq. (2.3) and solving for coefficients of different powers of $x^i t^j$ for a, p, q, we get $a = const., p = 0, q = -\frac{2aH}{b}$. Thus

$$u(x,t) = \frac{2H(2ax+b)}{-2Hat + ax^2 + bx + c}. \tag{3.4}$$

In what follows, Fig. 7 presents exact solution of Eq. (3.4) by choosing the suitable parameters.

4. Rogue wave solutions of Burgers' equation

To get the rogue wave solutions of Eq. (2.1), assuming $f(x,t)$ in the following new form $f = 1 + g^2 + h^2$,

$$f = 1 + g^2 + h^2 \tag{4.1}$$

with $g(x,t) = a_1 x + a_2 t + a_3$,

$$g(x,t) = a_1 x + a_2 t + a_3 \tag{4.2}$$

$$h(x,t) = a_4 x + a_5 t + a_6, \tag{4.3}$$

where a_i, $(1 \le i \le 6)$ are arbitrary constants to be determined later. Setting Eq. (4.1) into bilinear form Eq. (2.3), we obtain some polynomials which are functions of the variables x and t. Equating all the coefficient of x, t and the constant term to be zero, we can obtain the set of algebraic equations for a_i, $(1 \le i \le 6)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters a_i:

$$H = H, a_1 = a_1, a_2 = 0, a_3 = a_3, a_4 = a_1\sqrt{-1}, a_5 = 0, a_6 = a_6. \tag{4.4}$$

Therefore, substituting Eq. (4.4) and Eq. (4.1) along with Eq. (4.2) and Eq. (4.3) into Eq. (2.3) yields the following rogue wave solution,

$$u = \frac{2H\left\{2\left(xa_1 + a_3\right)a_1 + 2\left(-xa_1 + a_6\sqrt{-1}\right)a_1\right\}}{1 + \left(xa_1 + a_3\right)^2 + \left(xa_1\sqrt{-1} + a_6\right)^2}, \tag{4.5}$$

where the parameters satisfy the constraints (4.4). Fig. 8 shows the sketch of rogue waves for dissimilar values H, a_1, a_3, a_6.

5. Interaction solution between Rogue wave and solitary wave

To get the interaction phenomena between line rogue wave solution and solitary wave solutions of Eq. (2.1), assuming $f(x,t)$ as a quadratic function with exponential part, that is,

$$f = 1 + g^2 + h^2 + \lambda\exp(\gamma), \tag{5.1}$$

where g and h are defined by Eq. (4.2) and Eq. (4.3), and $\gamma(x,t) = k_1 x + w_1 t$,$k_1, w_1$ are the constant parameters which are determined later.

Substituting Eq. (5.1) into Eq. (2.3), with the aid of symbolic computation system Maple, we can obtain the following relations among parameters:

$$H = \lambda = a_3 = a_4 = a_6 = k_1 = const., a_1 = a_4\sqrt{-1}, a_2 = 0, a_5 = 0,$$
$$w_1 = -Hk_1^2. \tag{5.2}$$

Substituting Eq. (5.1) and Eq. (5.2) into Eq. (2.3) yields the following interaction solution

$$u = \frac{2H\left\{2a_4\left(-a_4 x + a_3\sqrt{-1}\right) + 2\left(xa_4 + a_6\right)a_4 + \lambda k_1 e^{-Htk_1^2 + xk_1}\right\}}{1 + \left(a_4 x\sqrt{-1} + a_3\right)^2 + \left(xa_4 + a_6\right)^2 + \lambda e^{-Htk_1^2 + xk_1}}. \tag{5.3}$$

In what follows, Fig. 9 presents exact solution of Eq. (5.3) by choosing the suitable parameters, which can show the interaction phenomena between solitary wave and rogue wave.

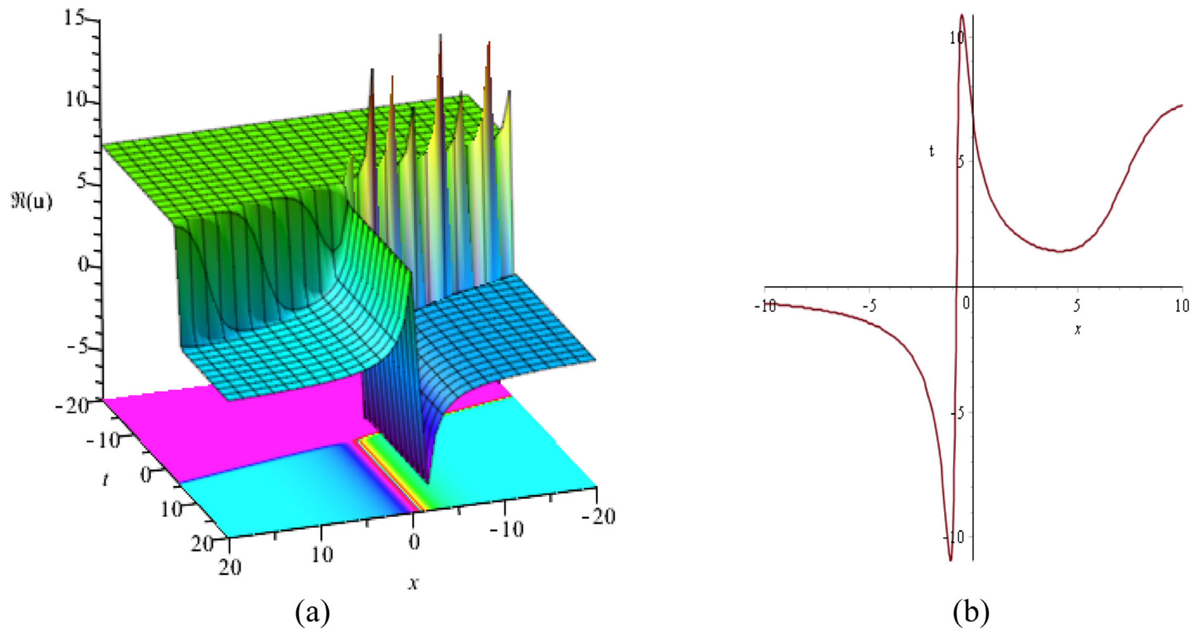

Fig. 9. Interaction of Line Rogue wave and kink solution (5.3) for Eq. (2.1) by choosing suitable parameters: $H = 3, \lambda = 1.5, a_3 = 1, a_4 = 2, a_6 = 3$ and $k_1 = 1.25$.(a) Perspective view of the wave and corresponding density plot (bottom) (b) The wave propagation pattern of the wave along the x axis.

6. Conclusion

In conclusion, the (1+1)-dimensional Burgers' equation has been investigated. We have derived solitary wave solutions and their interaction solutions Burgers' equation using the Cole–Hopf transformation. From the analysis we observed that when dispersive relation exists between the wave number and the wave speed with any one $a_{ij} = 0$or $a_{ijk} = 0$, then fission exist in the solution.

Based on the bilinear formalism, we also determine two classes of rational solutions, rogue wave solutions and rogue–kink wave solutions to the resulting equation. We observe that rational solution is worth checking if there exists a kind of Wronskian solutions to the Burgers' equation. We also conjecture that these two types of rational solutions in Eq. (3.3) and Eq. (3.4) would contain all rational solutions to the Burgers' equation. Meanwhile, the performances of the mentioned techniques are substantially powerful and absolutely reliable to search new explicit solutions of other NPDEs.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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