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Bootstrapping Unit Root Test for Small Sample: Application on Gross Domestic Product and Carbon Dioxide Emission in Bangladesh

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Abstract —*The power of conventional unit root tests is very low and suffers a severe size distortion problem in small sample time series. A simulation-based study is done to extract the performance of bootstrap on Augmented Dickey-Fuller (ADF) and Covariance Augmented Dickey-Fuller (CADF) tests. It is found that for small sample ($n = 30$) bootstrapped CADF test performs relatively better than all other tests. We perform these test on GDP per capita and Carbon Dioxide (CO_2) emission per capita of Bangladesh where data are available from 1972-2000 i.e. the data size is only 29. The test result shows that CO_2 emission per capita and GDP per capita in Bangladesh are unit root process. GDP per capita performs better as a covariate for applying CADF test on CO_2 emission in Bangladesh.*

Keywords and Phrases: Bootstrap, small sample, time series, unit root test.

JEL Classification: C01; C15; E27.

1. INTRODUCTION

Time series with unit root has great importance in the analysis of business and economics. It is frequently used to analyze the stability of business and economic measure over time. Tests of hypothesis of unit root in a time series process is yet challenging because the question whether a time series has a unit root is "inherently unanswerable on the basis of a finite sample observation" (Hamilton 1994) due to almost equal finite sample behavior of the processes with unit root and near unit root. The research on unit root test is extremely active in both theoretically and empirically. Different types of unit root tests are proposed and applied. Existing methods have some disagreements over how strong and extensive is the empirical evidence for the unit root in the process from which the sample is generated. The most frequently used method for the test of the presence of unit root in a parametric framework is the Dickey-Fuller test developed by Dickey and Fuller (1979, 1981). This test based on autoregression of finite order that is assumed to be known. In general, however, it is undesirable to test for the unit root within a specific parametric family, since misspecification could lead us to incorrect inference. Said and Dickey (1984) propose to include the lagged difference term for correction of serial correlation in the residuals proved that such Augmented Dickey-Fuller (ADF) tests are valid for all finite Autoregressive Moving Average (ARMA) processes of unknown order, if we increase the number of included lagged differences appropriately as the sample size gets large. However, several authors have argued that the tests may have considerable size distortions in finite samples, especially when the model has moving average components (Leybourne and Newbold 1999). Another test for testing unit root is Covariate Augmented Dickey-Fuller (CADF) test developed by Hansen (1995). He shows that inclusion of related stationary covariates in the regression of ADF equations might lead to a more precise estimate of the autoregressive coefficients. He also establishes that the asymptotic local power functions for the CADF *t*-statistics and shows that enormous power could be achieved by the inclusion of appropriate covariates. The major drawback of CADF test is that the limiting distribution of the CADF test statistic is dependent on the nuisance parameter that characterizes the measure of relative contribution of covariates to residuals of the equation without including covariates. So the critical values are based on an estimated nuisance parameter. Though relatively better from ADF test, size distortion problem is also present in CADF test for small sample realization. Chang and Park (2001) use sieve bootstrap method on ADF test for testing the unit root for infinite order autoregressive model. They show that the procedure improves the size distortion of ADF test for finite sample. In their simulation study, they took the initial value $y_0^* = 0$ which violates the stochastic properties of y_0^*. They used sample size 50 as small sample. Chang *et al.* (2001) used bootstrap method for CADF test that is independent of nuisance parameter. They show that the bootstrap CADF test was consistent and the critical values based on the empirical distribution of the test statistic obtained by bootstrap are asymptotically valid. They show their test performance by simulation. Results provide that CADF test offers drastic power gains over the conventional ADF test especially when the covariates are highly correlated with the error. Bootstrap CADF test significantly improves the finite sample size performances of the CADF test. In simulation their sample size was n = 50, 100 and 250. They claim that for n = 50 and 250 the results are "qualitatively similar".

In this article we have compared 4 different types of test. ADF, Bootstrapped ADF (BADF), CADF and bootstrapped CADF (BCADF). Potential problems with Chang *et al.* (2003) are that in real world situation we have sometime might not have enough data to generate a longer series of innovation to discard the 'burn-in' period observations. So, Chang *et al.* (2001) initialization method used for their bootstrapping technique is difficult for small sample non-stationary time series. Again Bootstrapping ADF test as was done by Chang and Park (2003) is also problematic for sample size of less than 50 since a number of lag difference term included in regression to whiten the residual. Sufficient inclusion of differenced lag term reduces the effective size of residuals.

The aim of the paper is to verify the result for sample size $n = 30$ with some correction of initialization to avoid the burn-in period by Chang *et al.* (2003). Stine's (1987) approach for initialization is used. The intuition is that, a random block from the main series will carry the properties of the series. So, a bootstrap replica of the series using such initial values, the generated series will preserve the properties of the original series. The important feature of Stine's (1987) initialization method is to backcast the initial using the backward dynamics properties of ARIMA process and then used the backcasted observation as the initial values. Stationary covariates are used for augmentation in the ADF model. It supports us to describe the unexplained serial correlation in the white noise of ADF model. For Bootstrapped ADF test sieve bootstrap was used. Finally we have decided to use the test procedure to test the unit root hypothesis for GDP per capita and CO_2 emission per capita of Bangladesh. The study is important in respect of Bangladesh because, the business and economic historical data documentation system is not frequently available and we sometimes get a short history data for analysis. This study provides sufficient evidence how confidently we can use bootstrap to test the presence of unit root with small sample data.

The layout of the article is as follows: the second section introduces the unit root problem in time series along with classical ADF and CADF test. Third section demonstrates the bootstrapping methods used to bootstrap ADF and CADF test while the fourth section describes the simulation algorithm and results, fifth section shows applications to the data series of GDP per capita and CO_2 emission per capita of Bangladesh. Section six concludes the study.

2. ADF AND CADF TESTS

The ADF test is proposed by Dickey and Fuller in 1981. In ADF test the hypothesis that the data is a unit root process, is tested in the context of three different maintained hypotheses or models concerning the alternative hypothesis that the series is stationary. These differ according to the presence of an intercept and/or deterministic trend. For the Augmented Dickey-Fuller test the following three models are considered:

$$\text{Model I} \quad \Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t$$

$$\text{Model II} \quad \Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t$$

$$\text{Model III} \quad \Delta y_t = \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t \tag{1}$$

Δ is the difference operator. u_t is assumed to be Gaussian white noise. Tests also are carried out on the parameters α and β. The DF test does not include the lag-differenced term. The purpose of including the lagged variables is to provide a correction for serial correlation that may be present in residual. When lagged dependent variables are included in the regression, the test is known as the Augmented Dickey-Fuller Test (ADF). In Model I the null is that y_t is a random walk process with drift while the alternative hypothesis is that y_t is a trend stationary process. In Model II the null is that y_t is a drift less random walk vs. a stationary process with mean α. Model III is a more powerful test for a unit root when the null is that y_t is a random walk with no drift against the alternative that y_t is a stationary process with mean zero. The test for a unit root is given by H_0; $\rho = 0$. If the series is I(0), against the hypothesis H_1; $\rho < 0$. To perform ADF test of a sample realization we first fit any one of the models (I, II or III) using OLS and estimate the test statistic τ. under H_0; $\rho = 0$, the test statistic is $\hat{\tau} = \hat{\rho}/\text{se}(\hat{\rho})$. Although the test statistic is calculated in the usual way, it's limiting distribution is Dickey-Fuller distribution rather than Gaussian. Statistical tables for Dickey-Fuller distribution for various sample size are reported in Hamilton (1994).

The CADF test is a unit root test procedure in multivariate context. Here more stationary variables are added to whiten the residuals from unexplained serial correlation in the models of ADF test. Let us assume that the regression residual u_t in model (I, II, III) in equation (1) are serially correlated and also allowed them to be related to other stationary covariates. Let w_t be the m-dimensional stationary covariates specified as;

$$\delta(L)u_t = \gamma(L)'w_t + \varepsilon_t \tag{2}$$

where L is the lag operator, and $\gamma(z) = \sum_{k=-r}^{q} \gamma_k z^k$ using equation (4) in equations (1) the AD$\delta(L) = 1 + \delta_1 L + \delta_2 L^2 + \cdots + \delta_p L^p$F models becomes

$$\text{Model I} \quad \Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \sum_{k=-r}^{q} \gamma_k' w_{t-k} + \varepsilon_t$$

$$\text{Model II} \quad \Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \sum_{k=-r}^{q} \gamma_k' w_{t-k} + \varepsilon_t$$

$$\text{Model III} \quad \Delta y_t = \rho y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \sum_{k=-r}^{q} \gamma_k' w_{t-k} + \varepsilon_t \tag{3}$$

We wish to test the hypothesis H_0 ; $\rho = 0$ against the alternative hypothesis H_1; $\rho < 0$. The long run covariance matrix

$$\Omega = \sum_{k=-\infty}^{\infty} E\left(\begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix}\begin{bmatrix} u_{t-k} & \varepsilon_{t-k} \end{bmatrix}\right) = \begin{pmatrix} \sigma_u^2 & \sigma_{u\varepsilon} \\ \sigma_{\varepsilon u} & \sigma_\varepsilon^2 \end{pmatrix} \tag{4}$$

$\lambda^2 = \sigma_{u\varepsilon}^2/\sigma_u^2\sigma_\varepsilon^2$ is a measures of relative contribution of w_t to u_t at the zero frequency. Under the null hypothesis the test statistic estimated by equation (7), $\tau(\hat{\rho}) = \hat{\rho}/\text{se}(\hat{\rho})$ has the distribution.

$$\tau(\hat{\rho}) \Rightarrow \lambda \frac{\int_0^1 W(r)dW(r)}{\left(\int_0^1 W^2(r)\right)^{\frac{1}{2}}} + (1-\lambda^2)^{\frac{1}{2}} N(0,1) \tag{5}$$

The asymptotic distribution depends on nuisance parameterλ^2. The critical values for the three models provided in the article of Hansen (1995) by Monte Carlo simulation for different values of λ^2.

3. Bootstrapping Time Series

In IID data structure one can create a bootstrap sample by random sampling with replacement. But it fails if data is time dependent due to heteroscedasticity or autocorrelation. In such situation if we use IID bootstrap, all the dependency information lost (Politis 2003). To preserve the dependency information, there are two approaches in resampling time series. 1) Parametric bootstrap and 2) Non-parametric bootstrap. Reviews of Barkoitz and Killian (1996) discussed both parametric and non-parametric approach in bootstrapping time series while In our article we use parametric bootstrap which seems best suited following Barkoitz and Killian (1996).

3.1 Bootstrapping ADF Test

To construct bootstrap ADF we first suppose that under the null hypothesis of unit root, $\Delta y_t = u_t$. Then we fit the autoregression model

$$\delta(L)u_t = \varepsilon_t \tag{6}$$

where ε_t is assumed to be Gaussian white noise. Usually, parameters of the model were estimated by Yule-Walker method, which always yields an autoregression that is invertible and it is asymptotically equivalent to the OLS method. Lag order is chosen by AIC. Basawa *et al.* (1991) showed that the samples generated without the unit root restriction do not behave like unit root processes. That makes the subsequent bootstrap techniques inconsistent. So it is important to base the bootstrap sampling on regression (6). The equation is the same as equation (1) Model III with the restriction $\rho = 0$. Then we obtain a bootstrapped sample ε_t^* from the centered residual and the bootstrapped realizations of u_t^* and y_t^* are obtained by the equation

$$\hat{\delta}(L)u_t^* = \varepsilon_t^* \tag{7}$$

For initial vector u_0^* we use Stine's (1997) approach of a random block selection from the series u_t of size p and

$$y_t^* = y_0^* + \sum_{k=1}^{t} u_k^* \tag{8}$$

that also requires the initial observation y_0^*. A random draw from the main sample realization is taken as the initial value of the bootstrapped sample y_t^*. Then ADF test is applied to y_t^* and we obtain The bootstrapped τ statistic $\hat{\tau}^* = \hat{\rho}^*/\text{se}(\hat{\rho}^*)$. The technique is repeated a number of times to obtain the empirical distribution of $\hat{\tau}$. At the nominal size 1%, 5% and 10% the critical value of bootstrapped test is the 1%, 5% and 10% empirical percentile of the empirical distribution. If the test statistic $\hat{\tau} \leq c^*(\alpha)$ the bootstrapped critical value of size α we reject the null hypothesis of a unit root.

3.2 Bootstrapping CADF Test

We have described the test statistic of CADF test $\tau(\hat{\rho})$ in section 2. We want to draw the bootstrapped critical value for $\tau(\hat{\rho})$ bootstrapping the sampling distribution of $\tau(\hat{\rho})$. To construct the bootstrap CADF test, letting $u_t = \Delta y_t$, Model III of equation (3) can be written in the form:

$$\hat{u}_t = \sum_{k=1}^{p} \hat{\delta}_k u_{t-k} + \sum_{k=-r}^{q} \hat{\gamma}_k' w_{t-k} + \hat{\varepsilon}_t \tag{9}$$

Equation (9) is estimated under the null hypothesis by OLS method and $\hat{\varepsilon}_t$ obtained and $\hat{\eta}_t$ from the l^{th} order autoregression for m dimensional covariate of the form

$$w_{t+r+1} = \hat{\Phi}_1 w_{t+r} + \cdots + \hat{\Phi}_t w_{t+r-l+1} + \hat{\eta}_t \tag{10}$$

The combined innovations are obtained from the equation (9) and equation (10) which were defined as $\hat{\xi}_t = (\hat{\varepsilon}_t, \hat{\eta}_t')'$. *1* + *m* dimensional bootstrap samples $\xi_t^* = (\varepsilon_t^*, \eta_t^{*\prime})'$ generated by re-sampling from the centered fitted residual vector $\left(\hat{\xi}_t - 1/n \sum_{t=1}^{n} \hat{\xi}_t\right)_{t=1}^{n}$. The bootstrapped sample of w_t^* obtained recursively from the fitted model

$$w_{t+r+1}^* = \hat{\Phi}_1 w_{t+r}^* + \cdots + \hat{\Phi}_l w_{t+r-l+1}^* + \eta_t^* \tag{11}$$

A new innovation is generated by

$$v_t^* = \sum_{k=-r}^{q} \hat{\gamma}_k w_{t-k}^* + \varepsilon_t^* \tag{12}$$

Using the innovation v_t^* we generate u_t^* by

$$u_t^* = \hat{\delta}_1 u_{t-1}^* + \hat{\delta}_2 u_{t-2}^* + \cdots + \hat{\delta}_p u_{t-p}^* + v_t^* \tag{13}$$

Finally y_t^* is generated by

$$y_t^* = y_0^* + \sum_{k=1}^{t} u_k^* \tag{14}$$

At all step, initial values are taken as was done in subsection 3.1. Using y_t^* and w_t^* we apply the classical CADF test and obtain the test statistic $\hat{\tau}^*$, the bootstrap replicate of $\hat{\tau}$ by

$$\tau^*(\hat{\rho}^*) = \frac{\hat{\rho}^*}{se(\hat{\rho}^*)} \tag{15}$$

$\hat{\rho}^*$ is the OLS estimate of ρ from the models of CADF test is applied to bootstrapped samples. To implement the bootstrap CADF tests, we repeat the bootstrap sampling for the given original sample a large number of times and obtain critical value $b_n^*(\alpha)$ such that α is the prescribed size. The bootstrap critical values are determined in the same manner as was done in the previous subsection 3.1. The bootstrap CADF tests reject the null hypothesis of a unit root if $\hat{\tau}(\rho) \leq b_n^*(\alpha)$

4. SIMULATION AND RESULTS

3.1 Simulation Algorithm

We wish to perform a set of simulation to investigate the relative performance of different (ADF, BADF, CADF, BCADF) unit root test. For simulation, we consider (y_t) as a unit root process of the form.

$$y_t = \rho y_{t-1} + u_t$$

The error term $u_t = y_t - y_{t-1}$ for $\rho = 1$ is generated by

$$u_t = \delta_1 u_{t-1} + v_t$$

where v_t is related to the covariate w_t by

$$v_t = \gamma w_t + \varepsilon_t$$

In our simulation the covariate w_t is assumed to follow AR (1) process of the form;

$$w_{t+1} = \varphi w_t + \eta_t$$

The innovations $\xi_t = (\varepsilon_t, \eta_t)'$ are iid. *N(0,Σ)* where

$$\Sigma = \begin{pmatrix} 1 & \sigma_{\varepsilon\eta} \\ \sigma_{\eta\varepsilon} & 1 \end{pmatrix}$$

Under this setup, we have the following covariate augmented Dickey-Fuller regression

$$\Delta y_t = \rho y_{t-1} + \alpha_1 \Delta y_{t-1} + \gamma w_t + \varepsilon_t$$

The correlation between v_t and w_t depends on two parameter values γ and φ. In the data generating process we use $\gamma = 0.8$ and $\varphi = 0.5$. The lagged difference term set to $\delta_1 = 0$ in our simulation. We want to test the hypothesis H_0; The process contains a unit root against the alternative hypothesis H_1; The series is a stationary process. To investigate the power and size of the test we considered ρ=0.5 and 0.95. The sample size taken n = 30. In our total simulation we took $\sigma_{\eta\varepsilon} = \sigma_{\varepsilon\eta} = 0.5$. After generating the data we performed 4 test procedures on the artificial data. To perform ADF test we use the most powerful model $y_t = \rho y_{t-1} + u_t$. Reason behind choosing such model is that in our DGP we generated y_t in the same fashion. We want to use ADF procedure taking the exact model. Next we used bootstrapped ADF test on the artificially generated data. Our model for bootstrapped ADF test is as follows;

$$\Delta y_t = \rho y_{t-1} + \sum_{k=1}^{p} \delta_k \Delta y_{t-k} + \varepsilon_t \tag{16}$$

The test statistic $\hat{\tau}$ is estimated from the model and it is compared to the tabulated value provided in Hamilton (1994).

For CADF test we used the model as follows;

$$\Delta y_t = \rho y_{t-1} + \delta_1 \Delta y_{t-1} + \gamma w_t + \varepsilon_t \tag{17}$$

The test statistic $\hat{\tau}$ and λ^2 were estimated from the model and they were compared to the tabulated value developed by Hansen (1995).

In bootstrapped CADF test we used model (17) for $\hat{\varepsilon}_t$ and the model

$$w_t = \sum_{i=1}^{l} \varphi_i w_{t-i} + \eta_t \tag{18}$$

for $\hat{\eta}_t$. Equation (18) is sometimes different from our DGP because after data generation we fitted AR model again to the artificial w_t using the Yule-Walker method for estimating the parameter. The maximum lag length is to be $10log_{10}(n)$. The default value of SPlus was 14.77 for sample size 30. We used AIC for lag order selection. 10,000 simulations were done to draw the conclusion. The power and size of the tests ADF, CADF, BADF and BCADF are illustrated in Figure 1 and Figure 2.

3.1 Simulation Result

Application of bootstrap technique does not improve the power of the test (Figure-1). In case of CADF test, the power of bootstrapped CADF is lower than classical CADF test at each prescribed level of significance. As level of significance increases, power of bootstrapped test is also increase. At 10% level of significance the power of both classical and bootstrapped tests are literally the same. In case of ADF test, we see that, the power of bootstrapped ADF test and classical ADF test are the same and like BCADF test, as the level of significance increases, the power of bootstrapped is also increase. Again if the residuals of ADF model are correlated with any other covariate or, if the model fails to explain the serial correlation completely, the power of ADF tests drastically falls. At that time the power of CADF and BCADF are much higher. These results support the result of Chang *et al.*(2001) for sample size $n = 50, 100$ and 250 under the same simulation setup.

Bootstrap techniques improve the size distortion problem of unit root rest for finite size of sample (Figure 2). Especially in bootstrapped CADF test reduce the size distortion which is amazing. The size distortion of both ADF and bootstrapped ADF tests are the same. Comparing the ADF and CADF tests the result shows that in the classical ADF and bootstrapped ADF tests, sizes distorts less than from both classical CADF and bootstrapped CADF test. This implies that bootstrap able to reduce size distortion. But the size distortion problem remains present in all of the tests considered in the article for sample of size$n = 30$. Chang *et al.* (2001) also found such type over rejection. Their over rejection rate was less, may be, due to their large sample size.

5. APPLICATION TO CO_2 EMISSION PER CAPITA AND GDP PER CAPITA

We apply all four tests to CO_2 emission per capita and GDP per capita of Bangladesh. Figure-3 and Figure-4 are the graphical presentation of the study series. The length of both the series is 29. Data are collected from *www.unep.org*, Geo-Dataset. At first classical tests ADF and CADF tests are applied. Models for these tests consist of constant and time trend. The null hypothesis H_0; the series is a unit root process with drift is tested against the alternative hypothesis H_1; the process is trend stationary. To determine the bootstrapped critical values of ADF and CADF test, Models are estimated under H_0 and these models are used as DGP (Data Generating Process) for bootstrap replicates of $\hat{\tau}$. We used CADF test to test the same hypothesis. ΔGDP and ΔCO_2 used as covariates for testing CO_2

emission per capita and GDP per capita respectively. The model for testing CO_2 emission consists of Δ GDP, one period leads of ΔGDP, one lag of ΔGDP, and two lags of ΔCO_2, Constant and time trend. The model for testing GDP data series consists of ΔCO_2, one lag of ΔCO_2, two lags of ΔGDP, constant and time trend. The model estimates are presented in Table 1-4.

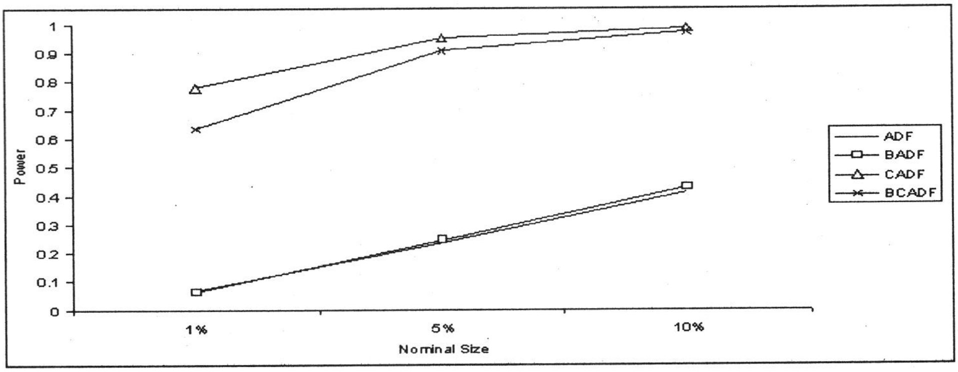

Figure 1. Power comparison of tests

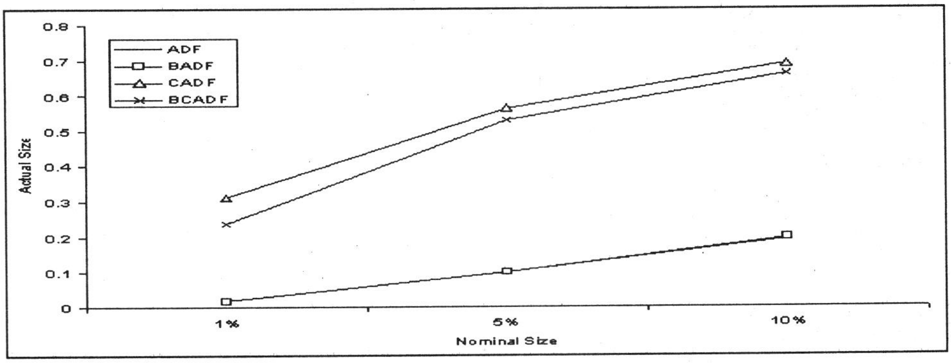

Figure 2. Trend CO_2 emission per capita of Bangladesh

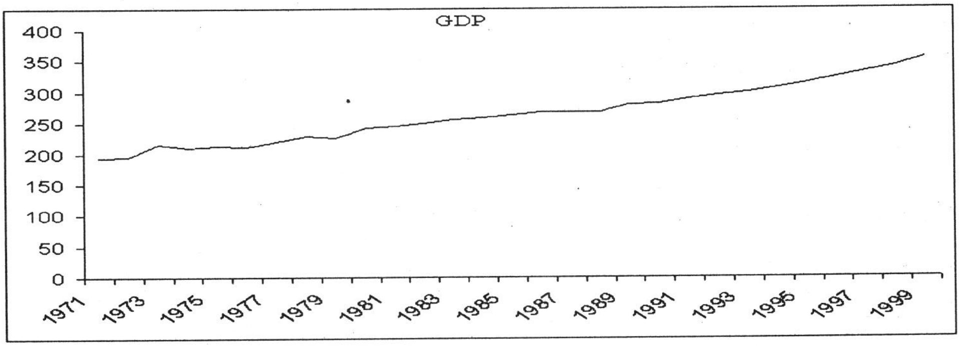

Figure 3. Size distortion of tests corresponding to their nominal size

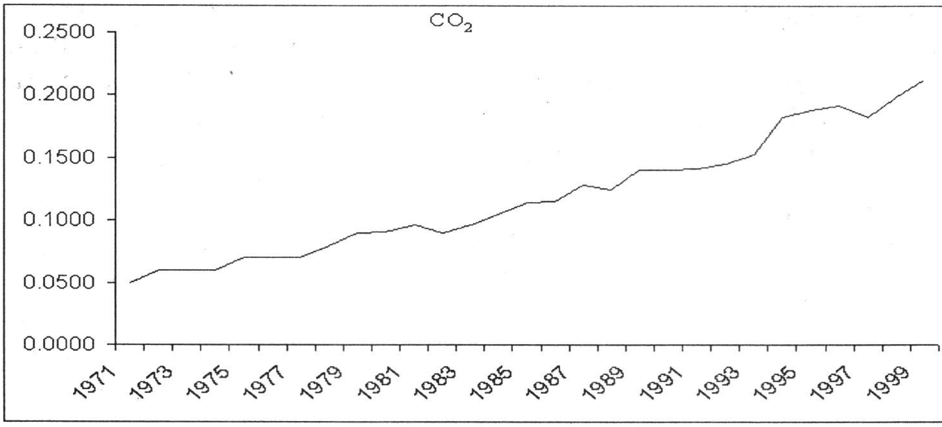

Figure 4. Trend of GDP per capita of Bangladesh

Table 1. Model Estimates for ADF Test and DGP for BADF Test for CO_2 Emission Per Capita

	Estimates	SE	Test Statistic	Model Under H_0
ρ	-0.444938655	0.203785644	-2.1833660	
C	0.022655397	0.009410052	2.4075739	0.0032173227
Trend	0.002705245	0.001136209	2.3809382	0 .0002626649
δ_1	0.034362297	0.214102542	0.1604946	0 .1727955335

Table 2. Model Estimates for ADF Test and DGP for BADF Test for GDP Per Capita

	Estimates	SE	Test Statistic	Model Under H_0
ρ	0.017340682	0.011102524	1.5618684	
C	-0.004535135	0.004208009	-1.0777390	3.31870845
Trend	0.255757718	0.153279584	1.6685700	0.34174135
δ_1	-0.437542807	0.172207222	-2.5407924	-0.42314426
δ_2	-0.041339108	0.169458140	-0.2439488	-0.03294488

Table 3. Model Estimates for CADF Test and DGP for BCADF Test for CO_2 Emission Per Capita

	Estimates	SE	Test Statistic	Model Under H_0
ρ	-0.5012830060	0.2247228011	-2.2306726	
C	0.0241944909	0.0097949570	2.4700967	0.00386278296
Trend	0.0029006816	0.0012098312	2.3975920	0.00025927046
δ_1	0.0761635767	0.2315364612	0.3289485	-0.18422440661
ΔGDP_t	0.0002655712	0.0004120174	0.6445631	-0.00000843938
ΔGDP_{t-1}	0.0001546688	0.0003252782	0.4754968	-0.00010027194

Table 4. Model Estimates for CADF Test and Data Generating Process for BCADF Test for GDP Per Capita Using Total Energy Consumption as Covariate

	Estimates	SE	Test Statistic	Model Under H_0
ρ	0.023142420	0.012829138	1.8038952	
C	0.005087908	0.004330911	-1.1747892	4.496682529
Trend	0.279509013	0.158488334	1.7635936	0.395094263
δ_1	0.474159612	0.179984758	-2.6344431	-0.457168450
δ_2	0.050960644	0.178401918	-0.2856508	-0.041346592
$\Delta(CO_2)_t$	0.002646021	0.002830244	-0.9349092	-0.002721990
$\Delta(CO_2)_{t-1}$	0.002055790	0.002829123	-0.7266527	-0.002132991

Table 5. Table of Critical Values for both Classical and Bootstrapped Tests

	CO_2			GDP		
λ^2	0.7864965			0.6577151		
Tests	1%	5%	10%	1%	5%	10%
ADF	4.3382	3.5867	-3.2279	-4.3382	-3.5867	-3.2279
BADF	-4.3319	-3.5934	-3.2078	-4.4785	-3.5102	-3.2235
CADF	-3.8241	-3.2554	-2.9599	-3.7242	-3.1433	-2.8342
BCADF	-4.6336	-3.8911	-3.4598	-0.8831	-0.3904	-0.0986

The critical values based on DF distribution, Bootstrapped Critical values, Critical values of CADF test based on estimated λ^2 and bootstrapped critical value of CADF test statistics are reported in Table-5. Result of Dickey-Fuller test shows that CO_2 emission in Bangladesh is a random walk process. Gaussianity assumption free ADF test based on bootstrapped critical value provides the result of ADF test. The CADF test and bootstrapped CADF test failed to reject the null hypothesis at 10% level of significance. Similarly ADF test and CADF test of GDP per capita do not reject the null hypothesis. The bootstrapped ADF and bootstrapped CADF test also provides the same result.

6. Conclusion

In this paper we have tried to know about the performance of bootstrapped ADF and bootstrapped CADF compared to ADF and CADF developed by Hansen (1995) for sample size 30 and newly introduce initialization method. 10,000 simulations have done. Analysis shows that bootstrap technique reduces size distortion problem for small sample compared to their classical ones. Bootstrapped ADF and bootstrapped CADF test does not achieve radical power gain over their classical ones but at higher level of significance, bootstrapped and classical test's power are almost the same. Result implies that for small sample time series, when there exists a unit root, bootstrapped version of both ADF and CADF tests can detect more frequently than the classical one. But, if a small time series realization that is generated from a stationary process, the bootstrapped ADF and CADF test equally likely rejects the null hypothesis as the classical do. Tests are applied on CO_2 emission per capita and GDP per capita of Bangladesh. All tests support that CO_2 emission per capita and GDP per capita are explained better by random walk process though the smooth increasing trend seems to us that both GDP and CO_2 emission per capita increasing rate is constant over time.

In business, economics and market study time series data frequently appears. Most of the time series researchers intend to project the future outcome by analyzing the trend and nature of the trend inherited in the dataset. Often researcher and analysts face the small sample realization problem. These are due to either they have not enough historical record or sometimes they like to draw inference quickly by getting a small sample within a limited time. The study concludes, bootstrap is better option to apply the unit root test on small sample time series data.

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