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Characteristics of the solitary waves and rogue waves with interaction phenomena in a (2 + 1)-dimensional Breaking Soliton equation

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ABSTRACT

Under inquisition in this paper is a (2 + 1)-dimensional Breaking soliton equation, which can describe various nonlinear scenarios in fluid dynamics. Using the Bell polynomials, some proficient auxiliary functions are offered to apparently construct its bilinear form and corresponding soliton solutions which are different from the previous literatures. Moreover, a direct method is used to construct its rogue wave and solitary wave solutions using particular auxiliary function with the assist of bilinear formalism. Finally, the interactions between solitary waves and rogue waves are offered with a complete derivation. These results enhance the variety of the dynamics of higher dimensional nonlinear wave fields related to mathematical physics and engineering.

1. Introduction

Nonlinear phenomena have a lot of significant applications in different sides of physics with natural and engineering fields. Basically all the fundamental equations of physics are nonlinear and, generally, such types of nonlinear evolution equations (NLEEs) are often very tough to solve clearly. The exact solutions of NLEEs play a crucial role in the study of nonlinear physical or natural phenomena. As a result, the powerful and effective methods to seek exact solutions of NLEEs still have attracted too much interest by a diverse group of scientists. To obtain the exact solutions of NLEEs, there are many powerful and systematic approaches have been developed, such as the inverse scattering transformation [1], the Darboux transformation [2], the tanh-function method [3], the extended tanh-function method [4], the homogeneous balance method [5], the Jacobi elliptic function expansion method [6], the F-expansion method [7], the variable separation method [8], Hirota bilinear method and so on [9–11]. Among those methods, the Hirota's bilinear method is rather heuristic and possesses significant features that make it practical for the determination of multiple soliton solutions, and for multiple singular soliton solutions [12–18] for a extensive class of NLEEs in a direct method. Recently, we have seen two types of phenomena, two or more solitons may fuse to a single soliton and at a specific time, a single soliton may fission to two or more solitons. These types of scenarios were called as soliton fission and soliton fusion respectively [19]. Indeed, people have observed these types of phenomena in many nonlinear science and engineering field such as the gas dynamics, laser, plasma physics, electromagnetic, and passive random walker dynamics [20–22]. Therefore, it very necessary to discuss about the elastic interactions into the solitary waves in certain integrable or non-integrable system with a strong physical backgrounds. Recently, rogue wave solutions have drawn a big attention of mathematicians and physicists globally for amusing class of lump-type solutions. Such types of phenomena are found in different fields in physics such as plasmas, the deep ocean, nonlinear optic and even finance [23–25]. In contrast to storms and tsunamis linked with cyclone which be able to predicated hours in advance [26]. On the basis of Hirota bilinear forms, it is natural and interesting to hunt for lump or lump-type solutions of NLEEs [27,28].

Besides the soliton solutions, Meng has presented some periodic solitary wave solutions for the soliton equations which are obtained by appropriate test functions with the help of bilinear form [30]. There are many researchers have been studied in Breaking

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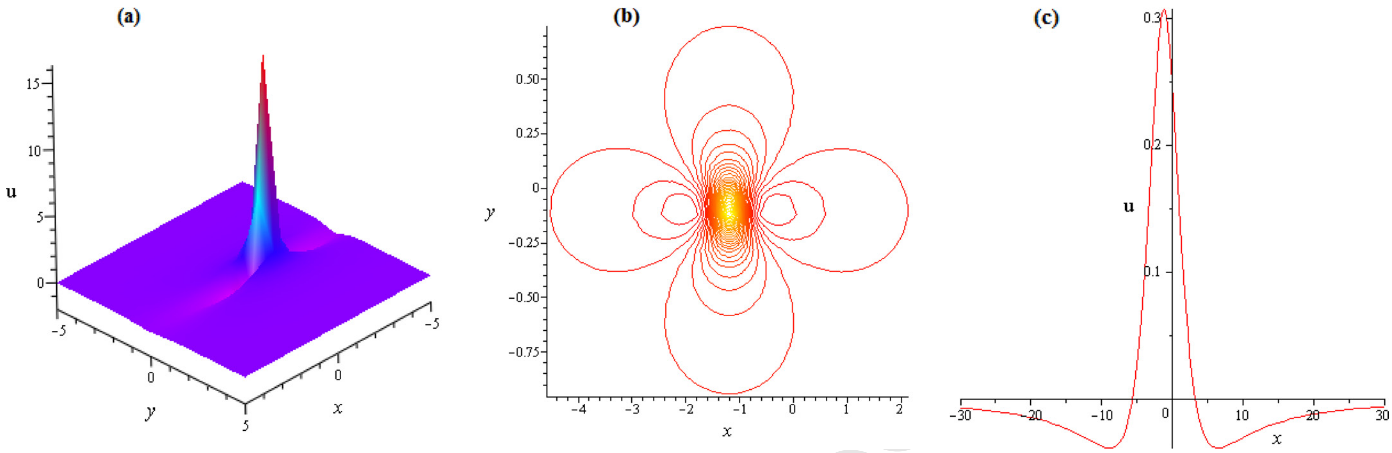

Fig. 1. (Color online.) Rogue wave solution (10) for Eq. (1) by choosing suitable parameters: $a_1 = -1$, $a_4 = -2$, $a_5 = 2$, $a_6 = -4$, and $a_8 = 2$. (a) Perspective view of the wave at $t = 10$. (b) Corresponding contour plot of the wave. (c) The wave propagation pattern of the wave along the x axis.

soliton equation (BSE) in many ways such as: Zhang formed non-traveling wave solutions to BSE by a generalized auxiliary equation method [31], Mei investigated general solution of BSE using the projective Riccati equation expansion method [32], Peng solved BSE by the singular manifold method [33], and Dai derived BSE chaotic behaviors by the mapping method [34]. The structures of $(2+1)$-dimensional BSE are rich and there are still more structures to be discovered.

In this paper, we will focus on the $(2+1)$-dimensional Breaking Soliton (BS) equation to show the diversity of such interaction solutions aid of symbolic computation with Maple. The $(2+1)$-dimensional BS equation has a Hirota bilinear form, and so, we will do a search for positive quadratic function solutions to the corresponding $(2+1)$-dimensional bilinear BS equation. The obtained quadratic function solutions contain a set of free parameters, and taking special choices of parameters involved.

2. Rogue and solitary wave solutions to the Breaking Soliton equation

2.1. The Breaking Soliton (BS) equation

The $(2+1)$-dimensional Breaking Soliton (BS) equation [29,31,32] reads as

$$P_{BS}(u,v) = u_t + \alpha u_{xxy} + 4\alpha u v_x + 4\alpha u_x v = 0, \tag{1}$$

where α is arbitrary constant and $u_y = v_x$. It is known that the BS equation above possesses a Hirota bilinear form:

$$\begin{aligned} B_{BS}(f) &:= \left(D_x D_t + \alpha D_y D_x^3\right)(f \cdot f) \\ &= \left[f_{xt} f - f_t f_x \right. \\ &\quad \left. + \alpha(f_{xxxy} f - 3 f_{xxy} f_x + 3 f_{xy} f_{xx} - f_y f_{xxx})\right] = 0, \end{aligned} \tag{2}$$

under the links from f to u and v are as follows:

$$u = 3(\ln f)_{xx} = \frac{3(f_{xx} f - f_x^2)}{f^2}, \tag{3}$$

and

$$v = 3(\ln f)_{xy} = \frac{3(f_{xy} f - f_x f_y)}{f^2}. \tag{4}$$

Such potential transformations used in Bell polynomial theories of soliton equations and a proper relation is

$$P_{BS}(u,v) = \left[\frac{B_{BS}(f)}{f^2}\right]_{xx}. \tag{5}$$

It is clear that, if f solves the bilinear breaking soliton equation (2), then $u = 3(\ln f)_{xx}$ and $v = 3(\ln f)_{xy}$ will solve the $(2+1)$-dimensional breaking soliton equation (1).

2.2. Rogue wave solutions

Let us adopt that Eq. (2) has a ansatz in the following form:

$$f = 1 + g^2 + h^2, \tag{6}$$

with

$$g(x,y,t) = a_1 x + a_2 y + a_3 t + a_4, \tag{7}$$

$$h(x,y,t) = a_5 x + a_6 y + a_7 t + a_8, \tag{8}$$

where a_i $(1 \le i \le 8)$ are arbitrary constants to be determined later. Setting Eq. (6) into bilinear form Eq. (2), we obtain some polynomials which are functions of the variables x, y and t. Equating all the coefficient of x, y, t and the constant term to be zero, we can obtain the set of algebraic equations for a_i $(1 \le i \le 8)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters a_i:

$$\begin{aligned} &a_1 = a_1, \quad a_2 = -\frac{a_5 a_6}{a_1}, \quad a_3 = 0, \quad a_4 = a_4, \quad a_5 = a_5, \\ &a_6 = a_6, \quad a_7 = 0, \quad a_8 = a_8. \end{aligned} \tag{9}$$

Therefore, substituting Eq. (9) and Eq. (6) along with Eq. (7) and Eq. (8) into Eq. (2) yields the following rogue wave solution,

$$\begin{aligned} u = &\frac{3(2a_1^2 + 2a_5^2)}{1 + (a_1 x - \frac{a_5 a_6 y}{a_1} + a_4)^2 + (a_5 x + a_6 y + a_8)^2} \\ &- \frac{3(2(a_1 x - \frac{a_5 a_6 y}{a_1} + a_4)a_1 + 2(a_5 x + a_6 y + a_8)a_5)^2}{(1 + (a_1 x - \frac{a_5 a_6 y}{a_1} + a_4)^2 + (a_5 x + a_6 y + a_8)^2)^2}, \end{aligned} \tag{10}$$

where the parameters satisfy the constraints (9). Fig. 1 shows the sketch of rogue waves for dissimilar values a_1, a_4, a_5, a_6, a_8, (a) gives 3D views from which one can reveal the standard rogue wave feathers. It is also clear that the Fig. 1 of Eq. (10) is the recognized eye-shaped rogue wave solution which has one local hump and two valleys (clears from the views (b)). Besides this, we discover that rogue wave has the uppermost peak in its surrounding waves and it can be forms in a short time and also can be realized from the perspective view of (c).

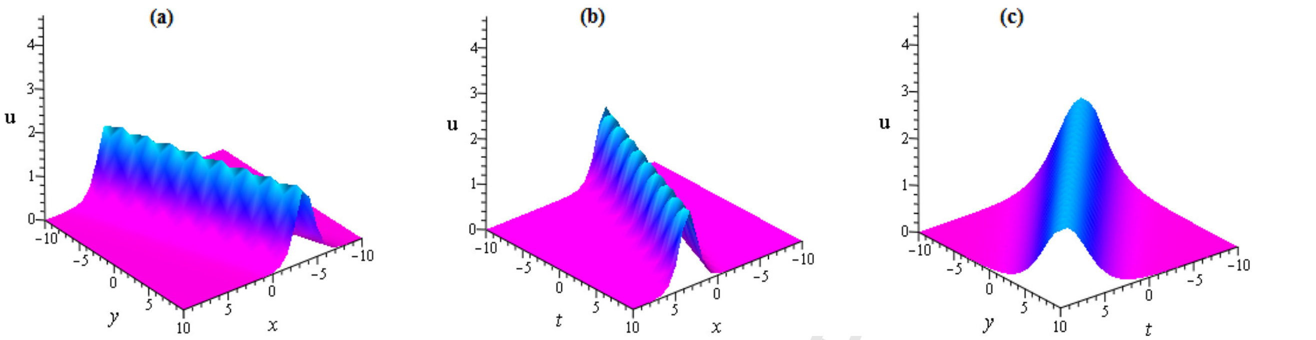

Fig. 2. (Color online.) One-soliton solution Eq. (17) for Eq. (1) by choosing suitable parameters: $k_1 = 1.5$, $k_2 = 0.5$, $\lambda = 1.5$, $\mu = 0.5$. (a) Perspective view of the wave at $t = 0$. (b) Perspective view of the wave at $y = 0$. (c) Perspective view of the wave at $x = 0$.

2.3. Solitary wave solutions

Here, we seek the solitary wave solutions of Eq. (1). We expand the test function f with small parameter λ

$$f(x, y, t) = 1 + \lambda f^{(1)} + \lambda^2 f^{(2)}, \tag{11}$$

with

$$f^{(1)} = \exp(k_1 x + k_2 y + k_3 t), \qquad f^{(2)} = \exp(k_4 x + k_5 y + k_6 t), \tag{12}$$

where k_i $(1 \le i \le 6)$ are arbitrary constants to be determined later. Setting Eq. (11) into bilinear form Eq. (2) and equating all the coefficient of exponential term to be zero, we can obtain the set of algebraic equations for k_i $(1 \le i \le 6)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters k_i:

$$k_1 = k_1, \quad k_2 = -\frac{k_1 k_5 (k_1 - 2k_4)}{k_4(2k_1 - k_4)}, \quad k_3 = -\frac{\mu k_1^3 k_5 (k_1 - 2k_4)}{k_4(2k_1 - k_4)},$$
$$k_4 = k_4, \quad k_5 = k_5, \quad k_2 = -\mu k_4^2 k_5. \tag{13}$$

Substituting Eq. (13) and Eq. (11) into Eq. (2) yields the following two-soliton solution

$$u = \frac{3(\lambda k_1^2 e^{\rho} + \lambda^2 k_4^2 e^{\rho_1})}{1 + \lambda e^{\rho} + \lambda^2 e^{\rho_1}} - \frac{(\lambda k_1 e^{\rho} + \lambda^2 k_4 e^{\rho_1})^2}{(1 + \lambda e^{\rho} + \lambda^2 e^{\rho_1})^2}, \tag{14}$$

with

$$\rho = k_1 x - \frac{k_1 k_5 (k_1 - 2k_4)}{k_4(2k_1 - k_4)} y + \frac{\mu k_1^3 k_5 (k_1 - 2k_4)}{k_4(2k_1 - k_4)} t, \tag{15}$$
$$\rho_1 = k_4 x + k_5 y - \mu k_4^2 k_5 t.$$

If we taking $f^{(2)} = 0$ in Eq. (11) and inserting Eq. (11) with Eq. (12) into bilinear form Eq. (2) and equating all the coefficient of exponential term to be zero, we can obtain the set of algebraic equations for k_i $(1 \le i \le 3)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters k_i:

$$k_1 = k_1, \quad k_2 = k_2, \quad k_3 = -\mu k_1^2 k_2. \tag{16}$$

Substituting Eq. (16) and Eq. (11) into Eq. (2) yields the following one-soliton solution

$$u = \frac{3\lambda k_1^2 e^{k_1 x + k_2 y - \mu k_1^3 k_2 t}}{1 + \lambda e^{k_1 x + k_2 y - \mu k_1^3 k_2 t}} - \frac{3\lambda^2 k_1^2 (e^{k_1 x + k_2 y - \mu k_1^3 k_2 t})^2}{(1 + \lambda e^{k_1 x + k_2 y - \mu k_1^3 k_2 t})^2}. \tag{17}$$

From Fig. 2, it is clear that the amplitude, velocity and width of the one-soliton keep constant during the wave propagation. One can show that the amplitudes of anxious position are limited and almost same in different spaces. In Fig. 3, the collision into the couple of bell-shaped soliton has elastic characteristics. When they fully meet, the amplitude changed and the changed amplitude is more than two times than the real amplitude of the two waves. The two waves converted to one eave direction after the collision with their original amplitude and shape. All the phenomena indicate that there is no energy loss during collision.

Now we will show the wave propagation situations of solitary wave by two figures. Fig. 2 and Fig. 3 show the one-soliton (17) and two-soliton solution (14), respectively, by choosing suitable parameters.

3. Interaction between rogue wave and solitary wave

In this section, we will be discussed the interaction phenomena between rogue wave solution and solitary wave solution of a $(2+1)$-dimensional breaking soliton equation. We choose two different cases of stripe soliton named exponential and hyperbolic sine function respectively.

3.1. First case

In the first case, we choose $f(x, y, t)$ as a quadratic function with exponential part, that is,

$$f = 1 + g^2 + h^2 + \lambda \exp(\gamma), \tag{18}$$

where g and h are defined by Eq. (7) and Eq. (8), and $\gamma(x, y, t) = k_1 x + k_2 y + k_3 t, k_i$ $(1 \le i \le 3)$ are the constant parameters which are determined later.

Substituting Eq. (18) into Eq. (2), with the help of symbolic computation system Maple, we get twenty equations in terms of unknown parameters. After, solving these equations, we find unknown parameters as:

$$\begin{cases} a_1 = const., \quad a_2 = -a_6\sqrt{-1}, \quad a_3 = 0, \quad a_4 = const., \\ a_5 = a_1\sqrt{-1}, \quad a_6 = const., \quad a_7 = 0, \\ a_8 = (a_4 - 1)\sqrt{-1}, \quad k_1 = const., \quad k_2 = \dfrac{1}{2}\dfrac{k_1 a_6\sqrt{-1}}{a_1}, \\ k_3 = -\dfrac{1}{2}\dfrac{\mu k_1^3 a_6 \sqrt{-1}}{a_1}. \end{cases} \tag{19}$$

Substituting Eq. (18) and Eq. (19) into Eq. (2) yields the following solution

$$u = \frac{3\lambda k_1^2 e^{\xi}}{1 + \xi_1^2 + \xi_2^2 + \lambda e^{\xi}} - \frac{3(2\xi_1 a_1 + 2\sqrt{-1}\xi_2 a_1 + \lambda k_1 e^{\xi})^2}{(1 + \xi_1^2 + \xi_2^2 + \lambda e^{\xi})^2}, \tag{20}$$

with

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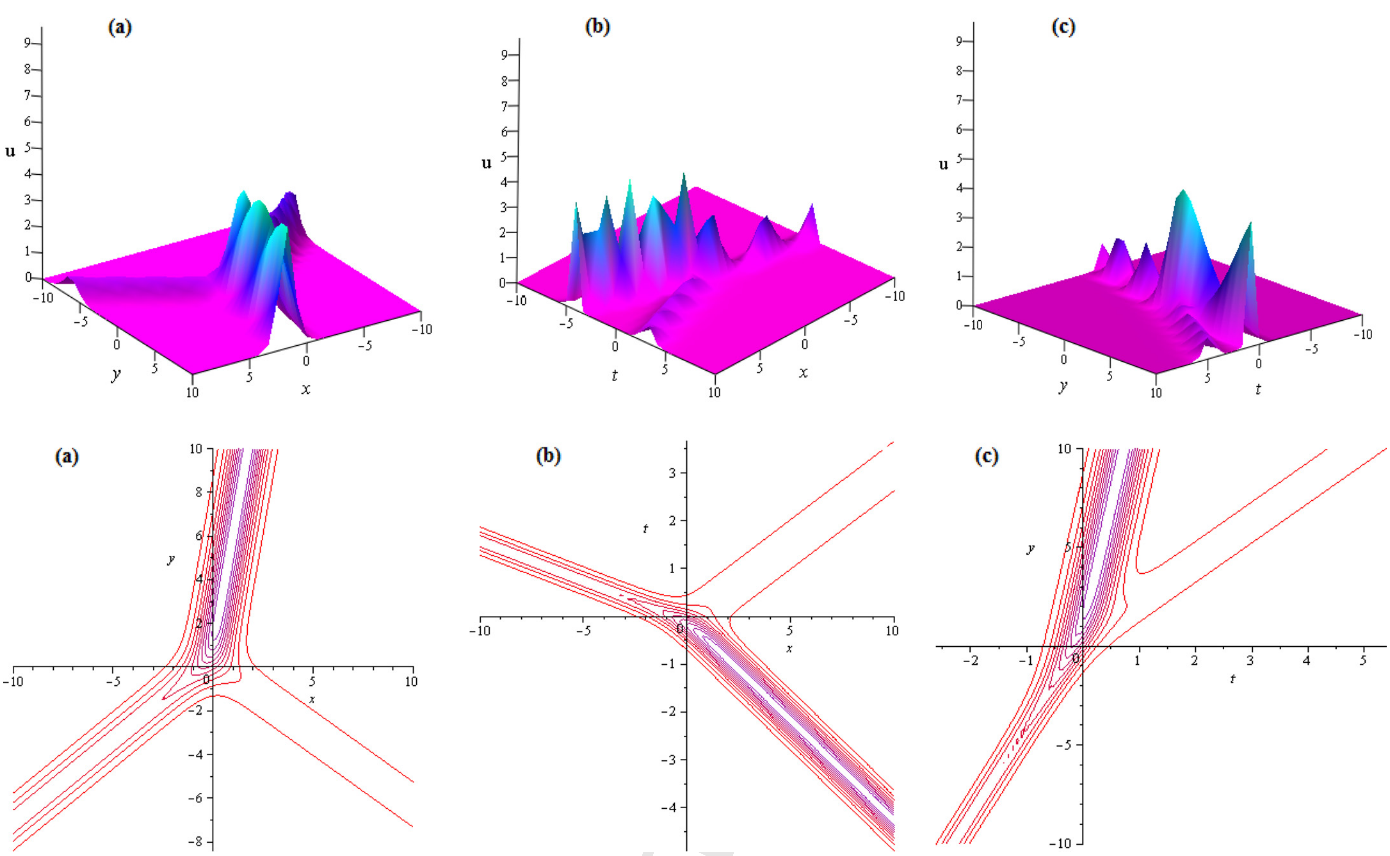

Fig. 3. (Color online.) Two-soliton solution Eq. (14) for Eq. (1) by choosing suitable parameters: $k_1 = 2.5$, $k_4 = 1.5$, $k_5 = -2$, $\lambda = 1.5$, $\mu = 2$: (a) Perspective view of the wave at $t = 0$. (b) Perspective view of the wave at $y = 0$. (c) Perspective view of the wave at $x = 0$ and corresponding contour plots (bottom) respectively.

$$\begin{cases} \xi = k_1 x + \dfrac{\frac{1}{2}\sqrt{-1}k_1 a_6 y}{a_1} - \dfrac{\frac{1}{2}\sqrt{-1}\mu k_1^3 a_6 t}{a_1}, \\ \xi_1 = a_1 x - \sqrt{-1}a_6 + a_4, \\ \xi_2 = \sqrt{-1}a_1 x + a_6 y + \sqrt{-1}(a_4 - 1), \end{cases} \tag{21}$$

where the parameters satisfy the constraints (19). In what follows, Fig. 4 and Fig. 5 appeared exact solution (20) by taking the suitable parameters.

3.2. Second case

Here, we choose $f(x, y, t)$ as a quadratic function with hyperbolic sine part, that is,

$$f = 1 + g^2 + h^2 + \lambda \sinh(\gamma), \tag{22}$$

where g, h and γ have been defined in the first case. Again, substituting Eq. (22) into Eq. (2), with the help of symbolic computation system Maple, gives the following equations for the parameters:

$$\begin{cases} a_1 = a_5\sqrt{-1}, \\ a_2 = -\dfrac{1}{3}\dfrac{a_7(-8\sqrt{-1}a_5^2 a_8^2 + 8\sqrt{-1}a_5^2 a_4^2 + 3\sqrt{-1}k_1^2\lambda^2 + 16a_5^2 a_8 a_4)}{\mu k_1^4 \lambda^2}, \\ a_3 = a_7\sqrt{-1}, \quad a_4 = a_4, \quad a_5 = a_5, \\ a_6 = \dfrac{1}{3}\dfrac{a_7(16\sqrt{-1}a_5^2 a_8 a_4 + 8a_5^2 a_8^2 - 8a_5^2 a_4^2 - 3k_1^2\lambda^2)}{\mu k_1^4 \lambda^2}, \\ a_7 = a_7, \quad a_8 = a_8, \quad k_1 = k_1, \quad k_2 = -\dfrac{4}{3}\dfrac{a_7 a_5(2\sqrt{-1}a_8 a_4 + a_8^2 - a_4^2)}{\mu k_1^3 \lambda^2}, \\ k_3 = \dfrac{4}{3}\dfrac{a_7 a_5(2\sqrt{-1}a_8 a_4 + a_8^2 - a_4^2)}{k_1 \lambda^2}. \end{cases} \tag{23}$$

Substituting these equations in g, h and γ which gives

$$\begin{cases} g(x, y, t) = a_5\sqrt{-1}x - \dfrac{1}{3}\dfrac{\psi_1}{\mu k_1^4 \lambda^2}y + a_7\sqrt{-1}t + a_4, \\ h(x, y, t) = a_5 x + \dfrac{1}{3}\dfrac{\psi_2}{\mu k_1^4 \lambda^2}y + a_7 t + a_8, \\ \gamma(x, y, t) = k_1 x - \dfrac{4}{3}\dfrac{\psi_3}{\mu k_1^3 \lambda^2}y + \dfrac{4}{3}\dfrac{\psi_3}{k_1 \lambda^2}t, \end{cases} \tag{24}$$

with

$$\begin{cases} \psi_1 = a_7\big(-8\sqrt{-1}a_5^2 a_8^2 + 8\sqrt{-1}a_5^2 a_4^2 \\ \qquad + 3\sqrt{-1}k_1^2\lambda^2 + 16a_5^2 a_8 a_4\big), \\ \psi_2 = a_7\big(16\sqrt{-1}a_5^2 a_8 a_4 + 8a_5^2 a_8^2 - 8a_5^2 a_4^2 - 3k_1^2\lambda^2\big), \\ \psi_3 = a_7 a_5\big(2\sqrt{-1}a_8 a_4 + a_8^2 - a_4^2\big). \end{cases} \tag{25}$$

Setting Eq. (24) into Eq. (22) along with Eq. (25), we obtain the expression of $f(x, y, t)$, which is

$$\begin{aligned} f = 1 &+ \left(a_5\sqrt{-1}x - \frac{1}{3}\frac{\psi_1}{\mu k_1^4 \lambda^2}y + a_7\sqrt{-1}t + a_4\right)^2 \\ &+ \left(a_5 x \frac{1}{3}\frac{\psi_2}{\mu k_1^4 \lambda^2}y + a_7 t + a_8\right)^2 \\ &+ \lambda \sinh\left(k_1 x - \frac{4}{3}\frac{\psi_3}{\mu k_1^3 \lambda^2}y + \frac{4}{3}\frac{\psi_3}{k_1 \lambda^2}t\right). \end{aligned} \tag{26}$$

Finally substitute Eq. (26) into Eq. (2), we obtain a new exact interaction solution of the $(2 + 1)$-dimensional breaking soliton equation

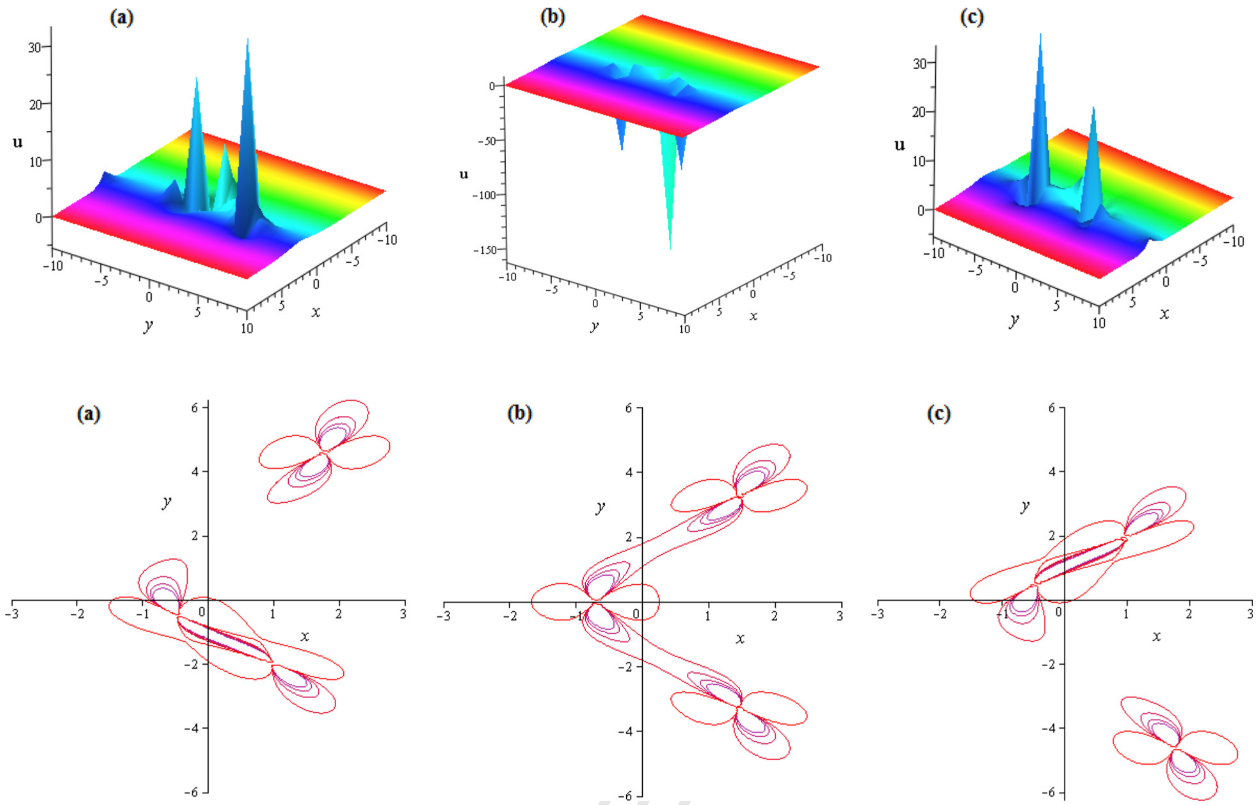

Fig. 4. (Color online.) Interaction phenomena between rogue wave and solitary wave solution (20) for Eq. (1) by choosing suitable parameters: $a_1 = 0.5$, $a_4 = 0.1$, $a_6 = -0.6$, $a_9 = 1$, $\lambda = 1$, $\mu = 2$ with 3D plots for different times (a) $t = -15$, (b) $t = 0$, and (c) $t = 15$ and corresponding contour plots (bottom) respectively.

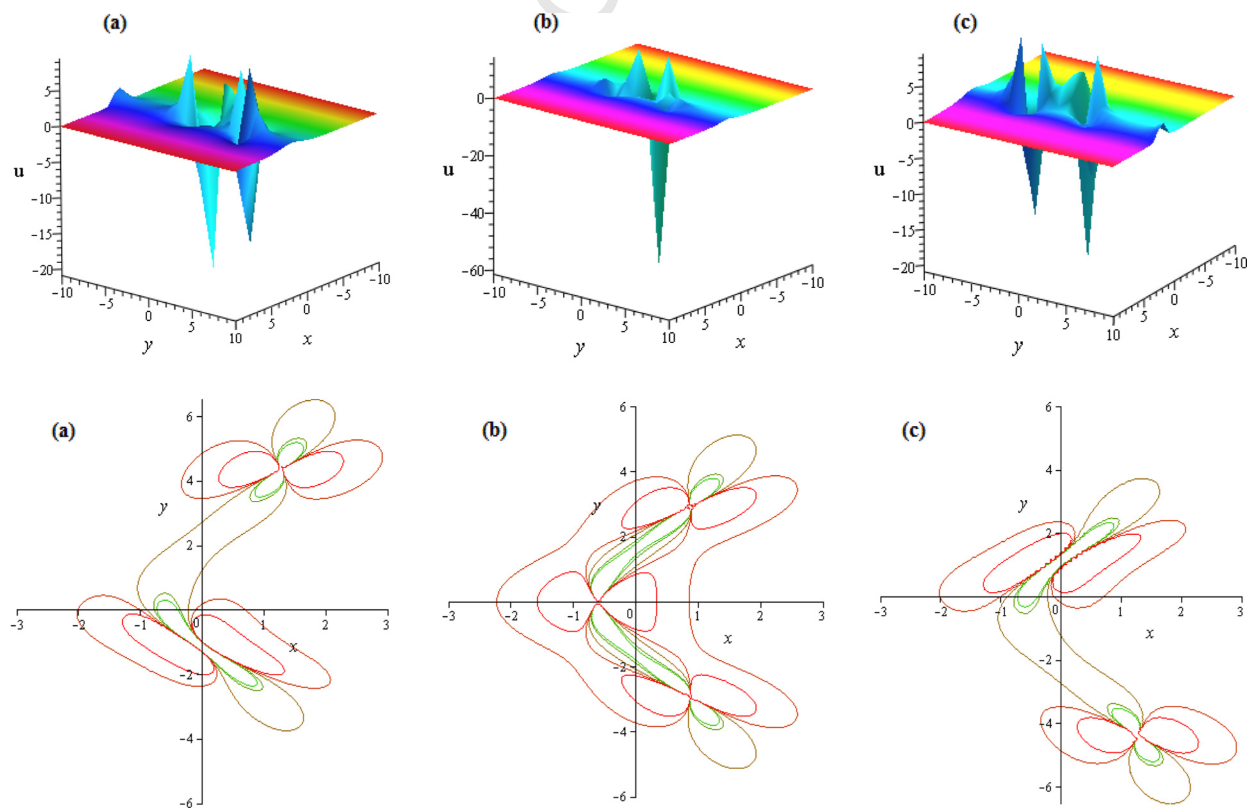

Fig. 5. (Color online.) Interaction phenomena between rogue wave and solitary wave solution (20) for Eq. (1) by choosing suitable parameters: $a_1 = 0.5$, $a_4 = -0.1$, $a_6 = -0.6$, $a_9 = 1$, $\lambda = 1.5$, $\mu = 2$ with 3D plots for different times (a) $t = -15$, (b) $t = 0$, and (c) $t = 15$ and corresponding contour plots (bottom) respectively.

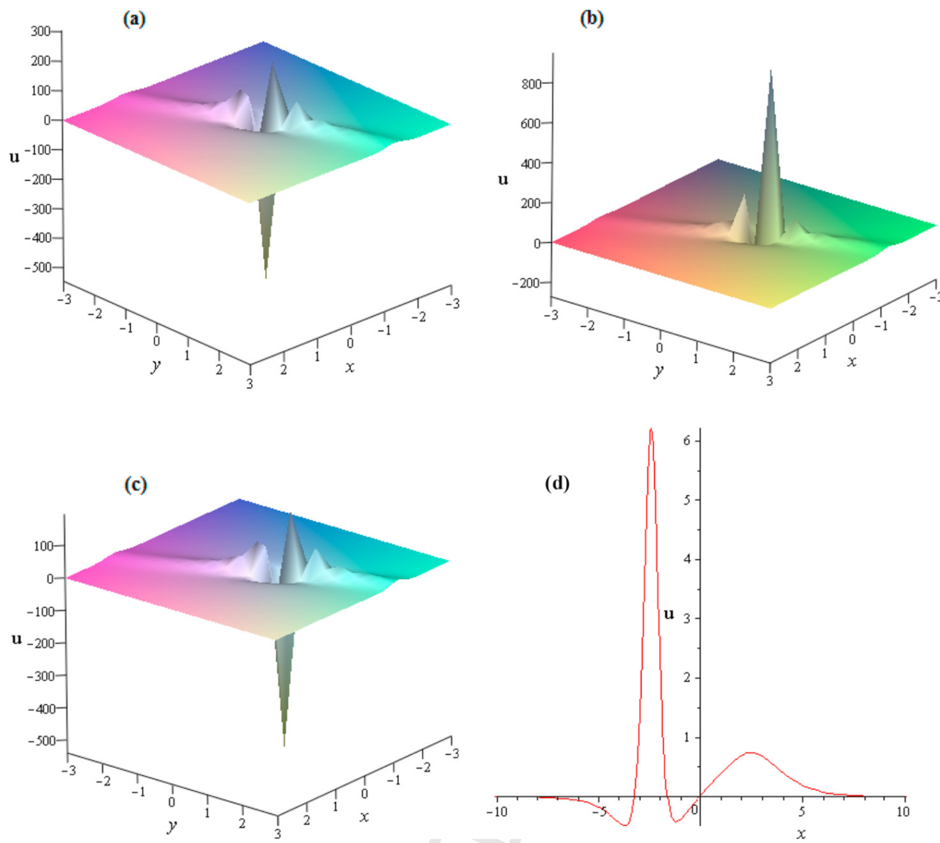

Fig. 6. (Color online.) Profile of interaction between rogue wave and hyperbolic solution (27) for Eq. (1) by choosing suitable parameters: $a_4 = 0.54$, $a_5 = -0.5$, $a_7 = 0.3$, $a_8 = -0.8$, $a_9 = 1$, $\lambda = 10$, $\mu = 0.1$, with 3D plots for different times (a) $t = 0$, (b) $t = 3$, and (c) $t = 5$ respectively; (d) 2D plot (c).

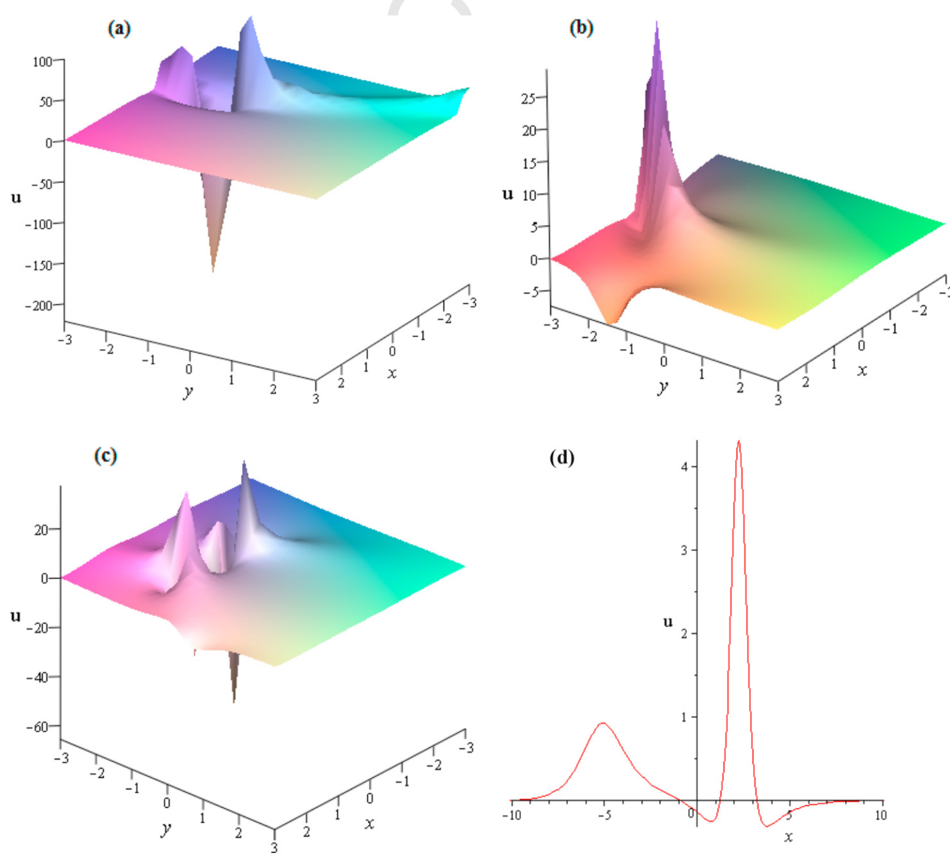

Fig. 7. (Color online.) Profile of interaction between rogue wave and hyperbolic solution (27) for Eq. (1) by choosing suitable parameters: $a_4 = 0.54$, $a_5 = -0.5$, $a_7 = 0.3$, $a_8 = -0.8$, $a_9 = 1$, $\lambda = 0.5$, $\mu = 2$ with 3D plots for different times (a) $t = 0$, (b) $t = 3$, and (c) $t = 5$ respectively; (d) 2D plot (c).

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$$u = \frac{-3\lambda k_1^2 \sinh \Delta_1}{(1+\Delta_2^2+\Delta_3^2+\lambda \sinh \Delta_1)} - \frac{3(2\sqrt{-1}\Delta_2 a_5 + 2\Delta_3 a_5 + \lambda k_1 \cosh \Delta_1)}{(1+\Delta_2^2+\Delta_3^2+\lambda \sinh \Delta_1)^2}, \tag{27}$$

with

$$\begin{cases} \Delta_1 = k_1 x - \dfrac{4}{3}\dfrac{\psi_3 y}{\mu k_1^3 \lambda^2} + \dfrac{4}{3}\dfrac{\psi_3 t}{k_1 \lambda^2}, \\ \Delta_2 = a_5\sqrt{-1}x - \dfrac{1}{3}\dfrac{\psi_2 y}{\mu k_1^4 \lambda^2} + \sqrt{-1}a_7 t + a_4, \\ \Delta_3 = a_5 x + \dfrac{1}{3}\dfrac{\psi_1 y}{\mu k_1^4 \lambda^2} + a_7 t + a_8, \end{cases} \tag{28}$$

where the parameters satisfy the constraints (23). In what follows, Fig. 6 and Fig. 7 appeared exact solution (27) by taking the suitable parameters.

4. Conclusions

In summary, based on the Hirota bilinear formulation and by a symbolic computation Maple, we have successfully presented some interaction phenomena between rogue waves and other kinds of solutions to the $(2+1)$-dimensional Breaking soliton equation. These results will serve as a very important milestone in the study of plasma physics and water waves phenomena. We also have demonstrated that the Hirota bilinear method is an effective tool to seek exact analytical solutions of other models in mathematical physics and engineering.

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