



Interactions of rogue and solitary wave solutions to the $(2 + 1)$ -D generalized Camassa–Holm–KP equation

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Abstract This research explores a $(2 + 1)$ -D generalized Camassa–Holm–Kadomtsev–Petviashvili model. We use a probable transformation to build bilinear formulation to the model by Hirota bilinear technique. We derive a single lump waves, multi-soliton solutions to the model from this bilinear form. We present various dynamical properties of the model such as one, two, three and four solitons. The double periodic breather waves, periodic line rogue wave, interaction between bell soliton and double periodic rogue waves, rogue and bell soliton, rogue and two bell solitons, two rogues, rogue and periodic wave, double periodic waves, two pair of rogue waves as

well as interaction of double periodic rogue waves in a line are established. Among the results, most of the properties are unexplored in the prior research. Furthermore, with the assistance of Maple software, we put out the trajectory of the obtained solutions for visualizing the achieved dynamical properties.

Keywords The gCHKP model · Hirota bilinear structure · Lump · Multi-soliton · Breather wave

Mathematical Subject Classification 35Q51 · 35Q53 · 35C99 · 37K10 · 74J35

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1 Introduction

Developing few precise results as well as new features to nonlinear partial evolution models (NLPEMs) is an important topic throughout soliton theories. In latest decades, researchers have been spending efforts to become skilled at solitonic solutions and interactions to NLPEMs and established several analytical approaches to extract such solutions, for instance the Darboux revolution [1], (G'/G) -expansion [2], exp-function [3, 4], sub-equation [5, 6], Hirota bilinear [7–9], bilinear network [10, 11] and Lie symmetry reductions [12] techniques. The Hirota bilinear technique is one of the major undeviating as well as expedient technique to achieve the precise solitonic elucidation to NLPEMs. When NLPEM be able to

reach in bilinear formulation, then the rogue, multi-soliton and more such type of breather wave solutions to the model are possible.

Last decades, scientists are greatly astonished to various dynamical signal solutions to their riveting category of rogue-lump solutions that be capable to originate in stream bearing wave of deep ocean [13, 14], plasma and nonlinear optics [15, 16] as well as Bose–Einstein solidifies [17]. Lou et al. [18] investigated lump wave solutions by means of variable separation technique in 2002. Few years ago, Ma and his co-investigators [19] recognized a common category of lump wave solution to the KPI equation. State shift lump propagations of $(2 + 1)$ -D gKdV model were investigated by Wang et al. [20]. Additional integral forms that preserved lump wave solutions are Boussinesq [21], BKP [22], Davey–Stewartson II [23], Ishimori-I [24] model equations. Due to coherent rationale results, lump propagation is analytical rational function solutions confined to a small area in every route in space whereas lump waves are limited to a small area in nearly every but not every route in space [25–28]. Rogue waves are bizarrely massive, erratic even rapidly occurring surface waves which can be immensely hazardous to hits the ships in deep ocean, even invert it due to massive ones. Rogue waves are confined in mutual space as well as time, arise from nowhere even vanish lacking a trace [29–32], have engaged with the liability of abundant oceanic disasters. More complicated phenomena and effects of nonlinearities can be described from multi-soliton solutions of NLPDEs [33, 34] using complex values to the free parameters involving in the multi-wave soliton solutions of the models [35, 36]. Peng [37] has investigated fractional-order chaotic system, while controlling of tentative nonlinear systems was studied by Gao et al. [38] and diagnosis of nonlinear systems was investigated by Rigatos et al. [39] with neural networks. Recently, enormous efforts have been salaried on diverge nonlinear models to derive collision among solitons, lump, rogue and hybrid solutions by dynamical researchers [40–42]. Motivated by these works in [40–42], we aim to derive such massive-type wave solution even more complicated nonlinear dynamical structures-like interactions of bell soliton with double periodic rogue waves, rogue with one and two bell soliton, double periodic waves as well as two pair of rogue waves in a line of the

gCHKP model [43]. Those solutions are still unexplored for any nonlinear models in the previous studies.

In this manuscript, we sport light on the $(2 + 1)$ -D gCHKP model [43] to detect distinguish behaviors of

$$(u_t + \alpha u_x + \beta u u_x + \gamma u_{xx})_x + u_{yy} = 0; \quad (1)$$

where α , β and γ are free invariables. Equation (1) is utilized to illustrate the task of spreading during creation of patterns inside fluid drops. In Ref. [43], authors have derived diverse wave structures as well as analysis stability of the solutions to the model. The various processes have been used to investigate the gCHKP model [Eq. (1)] such as sine–cosine as well as tanh method [44] and derived compact and non-compact wave solutions; homoclinic breather limit method [45] and derived breather, rogue and solitary waves solutions; Lie group [46], exp-function [47], ansatz [48] methods and presented single solitary as well as periodic wave solutions. Here, we are highly interested to establish collisions of bell soliton with double periodic rogue waves, rogue with bell soliton, rogue with two bell solitons, two rogues, double periodic waves as well as two pair of rogue waves in a line for the model.

The residual parts of manuscript are planned as: In Sect. 2, the bilinear structure to the gCHKP model is obtained by utilizing transformation $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$ and then a theorem is presented. We discuss lump surge propagations using positive quadratic function in next part 3. In part 4, the resulting N solitons are specified as well as various deformed waves with interactions are achieved. Conclusions are given with future task in the last part 5.

2 Bilinear formation of Camassa–Holm–KP model

Consider the probable renovation relation of potential transformation

$$u = c(t)q_{xx}, \quad (2)$$

where $c = c(t)$ indicates a constant. Inserting Eq. (2) interested in Eq. (1), yields

$$\begin{aligned} c(t)q_{3x,t} + \alpha c(t)q_{4x} + \beta \{c^2(t)q_{2x}q_{4x} + c^2(t)q_{3x}^2\} \\ + \gamma c(t)q_{5x,t} + c(t)q_{2x,2y} \\ = 0. \end{aligned} \quad (3)$$

Integrating one time with respect to x , setting integral constant to be zero, one obtains

$$q_{2x,t} + \alpha q_{3x} + \beta c(t) q_{2x} q_{3x} + \gamma q_{4x,t} + q_{x,2y} = 0. \quad (4)$$

Again integrating one time with respect to x , setting integral constant to be zero, we reach to

$$q_{x,t} + \alpha q_{2x} + \beta c(t) q_{2x}^2 + \gamma q_{3x,t} + q_{2y} = 0. \quad (5)$$

If one selects $c(t) = 6\gamma/\beta = \text{const.}$ as $q = 2 \ln f$ and utilizes formula Appendix (A10) in Ref.[49], the resulting Eq. (5) for q reads

$$E(q) = q_{xt} + \alpha q_{xx} + 6\gamma q_{2x}^2 + \gamma q_{xxx} + q_{yy} = 0. \quad (6)$$

Depending on results of P -polynomial which is offered in [49, 50] and utilizes formula Appendix (A10) in Ref. [49], one attains

$$E(q) = p_{xt}(q) + \alpha p_{xx}(q) + \gamma p_{xxx}(q) + p_{yy}(q) = 0. \quad (7)$$

The expression [Eq. (7)] of the gCGKP leads to bilinear presentation as

$$(D_x D_t + \alpha D_x^2 + \gamma D_x^3 D_t + D_y^2) f \cdot f = 0, \quad (8)$$

through mappings

$$q = 2 \ln f \Leftrightarrow u = c(t) q_{xx} = \frac{12\gamma}{\beta} (\ln f)_{xx}, \quad (9)$$

where f is a function of x, y, t .

Analyzing the beyond, we ascertain a theorem as follows:

Theorem The gCHKP Eq. (1) can be expressed as bilinear form.

$$(D_x D_t + \alpha D_x^2 + \gamma D_x^3 D_t + D_y^2) f \cdot f = 0, \quad (10)$$

$$\text{by the relation } u = \frac{12\gamma}{\beta} [\ln f(x, y, t)]_{xx}, \quad (11)$$

iff $6\gamma/\beta$ is a constant.

3 Rogue-type solution of gCHKP equation

To explore quadratic polynomial solutions of the (2 + 1)-D bilinear gCHKP model in (10), we initiate with

$$f = g^2 + h^2 + a_9, \quad (12)$$

in which

$$\begin{cases} g(x, y, t) = a_1 x + a_2 y + a_3 t + a_4 \\ h(x, y, t) = a_5 x + a_6 y + a_7 t + a_8 \end{cases}, \quad (13)$$

where a_i , ($1 \leq i \leq 9$) are free invariables to be calculated. Inserting Eq. (12) to bilinear structure equation (10) gives several polynomials in terms of x, y, t . Collecting coefficients of x, y, t as well as like invariable to be zero leads to a system of constraints in terms of a_i , ($1 \leq i \leq 9$), and solving the constraints by Maple software yields:

$$\begin{cases} a_1 = a_2 = a_4 = a_5 = a_6 = a_8 = \text{const.}, \\ a_3 = -\chi_1, a_7 = \frac{-\chi_1(a_1^3 + a_1 a_5^2) + \chi_2}{a_1^2 a_5 + a_5^3}, \\ a_9 = -\frac{6\{\chi_1(a_1^3 + a_1 a_5^2)(\gamma - 1) - \gamma \chi_2\}}{-a_5 a_7 + \alpha(a_1^2 + a_5^2) - a_1 \chi_1 + a_6^2 + a_2^2}, \end{cases} \quad (14)$$

where $\chi_1 = \frac{\alpha a_1^3 + \alpha a_1 a_5^2 + a_1 a_5^2 - a_1 a_6^2 + 2a_2 a_5 a_6}{a_1^2 + a_5^2}$ and $\chi_2 = (-a_1^2 + a_5^2)a_6^2 + 4a_1 a_2 a_5 a_6 + (a_1^2 - a_5^2)a_2^2 + \alpha a_1^4 + 2\alpha a_1^2 a_5^2 + \alpha a_5^4$.

Therefore, substituting Eq. (14) into Eq. (12) with Eq. (13) and transformation equation (11) yields most wanted rogue wave solution to Eq. (1). We promised that any specific time t , the lump wave result tends to zero, iff $g^2 + h^2 \rightarrow \infty$, i.e., the same as to $x^2 + y^2 \rightarrow \infty$.

At most or at least point in the lump wave, i.e., central point taking $t = t_0$ is

$$\begin{cases} x = \frac{a_2 a_7 t_0 - a_3 a_6 t_0 + a_2 a_8 - a_4 a_6}{a_1 a_6 - a_2 a_5}, \\ y = \frac{a_1 a_7 t_0 - a_3 a_5 t_0 + a_1 a_8 - a_4 a_5}{a_1 a_6 - a_2 a_5}, \end{cases} \quad (15)$$

where $a_1 a_6 - a_2 a_5 \neq 0$. Inserting Eq. (15) into Eq. (11), the maximum height of rogue reached at $\frac{24\gamma(a_1^2 + a_5^2)}{a_9 \beta} (a_9, \beta \neq 0)$, as of which anyone can detect that height of the rogue wave is dependent on a_1, a_5, a_9, β and γ .

Figure 1 shows the sketch of the 3-D view of rogue wave result to Eq. (1) in distinct planes, due to some values $\alpha = \beta = \gamma = 1$, $a_1 = a_4 = 2$, $a_2 = a_6 = 1$, $a_5 = -1$ and $a_8 = -2$. It is observed that Fig. 1

looks like eye-shaped rogue profile that consisted of two valleys as well as one confined hump. Besides this, it is noticed that rogue has upmost crest comparing with surrounding as well as it can be appearances within a short time.

4 Multi-soliton solutions and various deformed wave structures

This section is going to present the solitary wave solutions analytically from the bilinear form. We utilized the Hirota scheme to derive the N -soliton solution of the model from its bilinear representation [Eq. (10)]. Moreover, we ascertain rational solutions of model through a long wave limit technique. Consider f has multiple solitons with the structure

$$f = f_N = \sum_{\mu=0,1} \exp \left(\sum_{i=1}^N \mu_i \sigma_i + \sum_{i < j}^N \mu_i \mu_j B_{ij} \right). \quad (16)$$

4.1 Two solitons and their deformed structures into rogue and periodic waves

Owing to $N = 2$, f_2 takes the structure

$$f_2 = 1 + \exp \sigma_1 + \exp \sigma_2 + B_{12} \exp(\sigma_1 + \sigma_2), \quad (17)$$

where

$$\sigma_i = k_i(x + p_i y + q_i t) + \sigma_i^0, \quad (18)$$

Replace Eq. (17) into Eq. (11) in addition to a few over-simplifications, then linked with every part of the coefficients of $\exp$ to zero, one can gain solution as:

Including Eqs. (17)–(19) into Eq. (10) yields two-soliton solution to Eq. (1). Putting $\alpha = -0.005$, $\beta = 1$, $\gamma = 0.5$, $k_1 = k_2 = 2$, $p_1 = 1$, $p_2 = 2$, and $\sigma_1^0 = \sigma_2^0 = 0$, one be capable to achieve a 2-bell solitons solution that is depicted in Fig. 2a. Stand on over technique [Eq. (17)] presents breather as a result of picking proper invariables. Breather waves to Eq. (1) are possible in (x, y) , (x, t) as well as (y, t) plane from Eq. (19), under the parametric conditions

$$k_1 = Ib_1, \quad k_2 = -Ib_2, \quad p_1 = b_1, \quad p_2 = -b_1. \quad (20)$$

For instance, setting parameters as follows.

$k_1 = k_2^* = 2I$, $p_1 = -p_2 = 2$, $\alpha = 2$, $\beta = 4$, $\gamma = 1$, $\sigma_1^0 = \sigma_2^0 = 0$, we obtain breather waves and its outlines in distinct planes (Fig. 2b–d). This expression [Eq. (17)] can also be written as.

$$f = 1 + 2 \exp(4Iy) \cos(2x + 4t) + 7 \exp(8Iy),$$

and thus

$$u = \frac{12\gamma}{\beta} [\ln(1 + 2 \exp(4Iy) \cos(2x + 4t) + 3 \exp(8Iy))]_{xx}. \quad (21)$$

The shapes of the solution [Eq. (21)] have bullet-like double periodic waves in Fig. 3e. But when we set the constants as $\alpha = 2$, $\beta = 4$, $\gamma = 1$, $k_1 = k_2^* = 2I$, $p_1 = p_2^* = 2I$, $\sigma_1^0 = \sigma_2^0 = 0$, then Eq. (17) reduces to

$$q_i = -\frac{p_i^2 + \alpha}{\gamma k_i^2 + 1} (i = 1, 2),$$

$$B_{12} = \frac{\gamma^2 + \left\{ (3k_1^2 - 3k_1 k_2 + 2k_2^2)p_1^2 + (2k_1^2 - 3k_1 k_2 + 3k_2^2)p_2^2 - 2(k_1^2 + k_2^2)p_1 p_2 + 3\alpha(k_1^2 - k_2^2) \right\} \gamma + (p_1 - p_2)^2}{\left\{ (k_1^4 - 3k_1 k_2^3 + 3k_1^2 k_2^2)p_1^2 + (k_1^4 - 3k_1^3 k_2 + 3k_1^2 k_2^2)p_2^2 - 2k_1^2 k_2^2 p_1 p_2 + \alpha \left(\begin{array}{l} k_1^4 - 3k_1^3 k_2 + 4k_1^2 k_2^2 \\ -3k_1 k_2^3 + k_2^4 \end{array} \right) \right\} + \gamma^2 + \left\{ (3k_1^2 + 3k_1 k_2 + 2k_2^2)p_1^2 + (2k_1^2 + 3k_1 k_2 + 3k_2^2)p_2^2 - 2(k_1^2 + k_2^2)p_1 p_2 + 3\alpha(k_1^2 + k_2^2) \right\} \gamma + (p_1 - p_2)^2}.$$

(19)

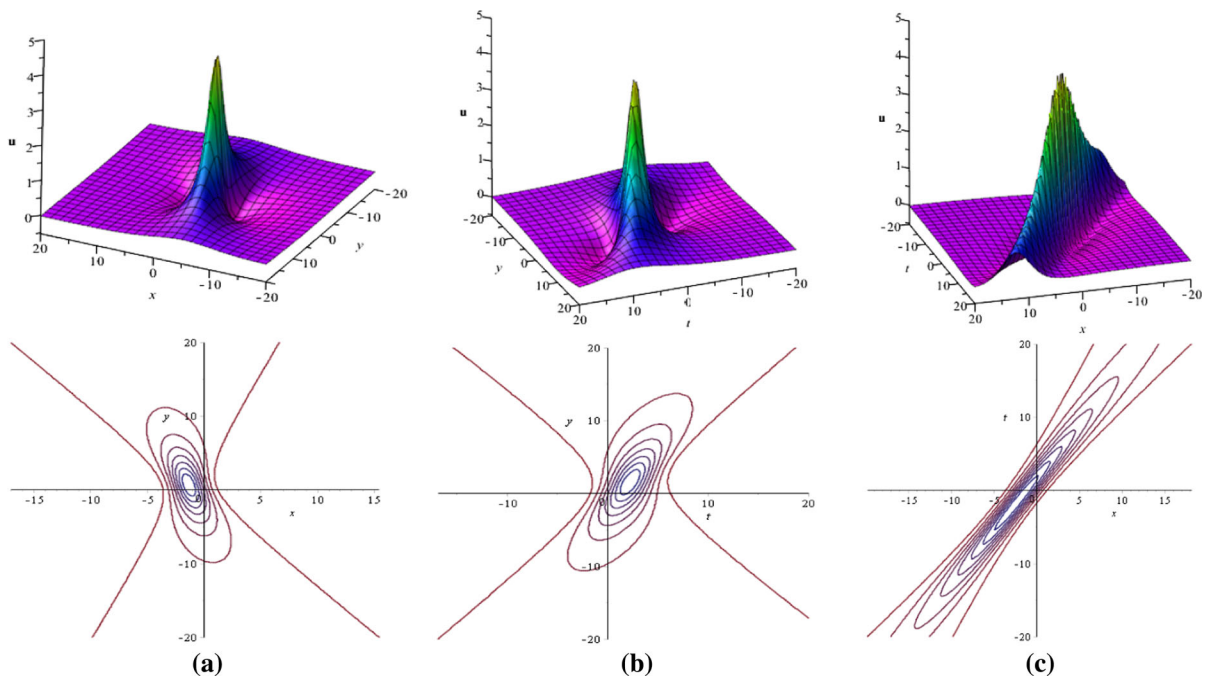


Fig. 1 Rogue solution (14) to Eq. (1) via selecting proper values: $\alpha = \beta = \gamma = 1$, $a_1 = a_4 = 2, a_2 = a_6 = 1, a_5 = -1$ and $a_8 = -2$, **a** outlook of the wave for $t = 0$; **b** outlook of

the wave for $x = 0$; **c** outlook of the wave for $y = 0$, as well as analogous contour profiles (below) correspondingly

completely real-valued and $f = 1 + 2 \exp(-4y) \cos(2x - 4t/3) + 3 \exp(-8y)$ and thus

$$u = \frac{12\gamma}{\beta} [\ln(1 + 2 \exp(-4y) \cos(2x - 4t/3) + 3 \exp(-8y))]_{xx}. \quad (22)$$

The profile of the solution [Eq. (22)] provides us periodic line rogue waves depicted in Fig. 3f.

Owing to uncover coherent elucidation to Eq. (10), a long limit of f in Eq. (17) have to exercise. For the purpose, set the parameters as

$$k_1 = l_1 \varepsilon, k_2 = l_2 \varepsilon, \sigma_1^0 = \sigma_2^{0*} = I\pi, \quad (23)$$

to Eq. (17) with captivating limit as $\varepsilon \rightarrow 0$, then f be able to write:

$$f = (\theta_1 \theta_2 + \theta_0) l_1 l_2 \varepsilon^2 + O(\varepsilon^3), \quad (24)$$

$$\begin{aligned} \text{where } \theta_i &= (p_i^2 + \alpha)t - y p_i - x, \theta_0 \\ &= -\frac{6\gamma(p_1^2 + p_2^2 + 2\alpha)}{(p_1 - p_2)^2}. \end{aligned} \quad (25)$$

Plugging Eqs. (24), (25) interested in Eq. (10), resulting u yields

$$u = -\frac{12\gamma}{\beta} \frac{(\theta_1^2 + \theta_2^2 - 2\theta_0)}{(\theta_1 \theta_2 + \theta_0)}. \quad (26)$$

We cover the rogue wave shapes for constants $p_1 = 2, p_2 = -2$ (Fig. 3). Evidently, Fig. 3 gratifies usual stripe rogue waves result.

4.2 Three solitons and their deformed structures into rogue, periodic and soliton by collisions

For $N = 3$, f_3 takes the structure

$$\begin{aligned} f_3 &= 1 + \exp \sigma_1 + \exp \sigma_2 + \exp \sigma_3 \\ &+ B_{12} \exp(\sigma_1 + \sigma_2) + B_{13} \exp(\sigma_1 + \sigma_3) \\ &+ B_{23} \exp(\sigma_2 + \sigma_3) + B_{12} B_{23} B_{13} \exp(\sigma_1 + \sigma_2 + \sigma_3), \end{aligned} \quad (27)$$

with

$\sigma_i = k_i(x + p_i y + q_i t) + \sigma_i^0$, $i = 1, 2, 3$ and q_i, B_{ij} in which $i < j; i, j = 1, 2, 3$ comes from Eq. (16).

The values of $\alpha = -0.005, \beta = 1, \gamma = 0.5$, $k_1 = k_2 = k_3 = 2, p_1 = 1, p_2 = 2, p_3 = -1$ and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = 0$ yield interaction of three bell solitons solution which is depicted in Fig. 4a. Figure 4a shows that it is

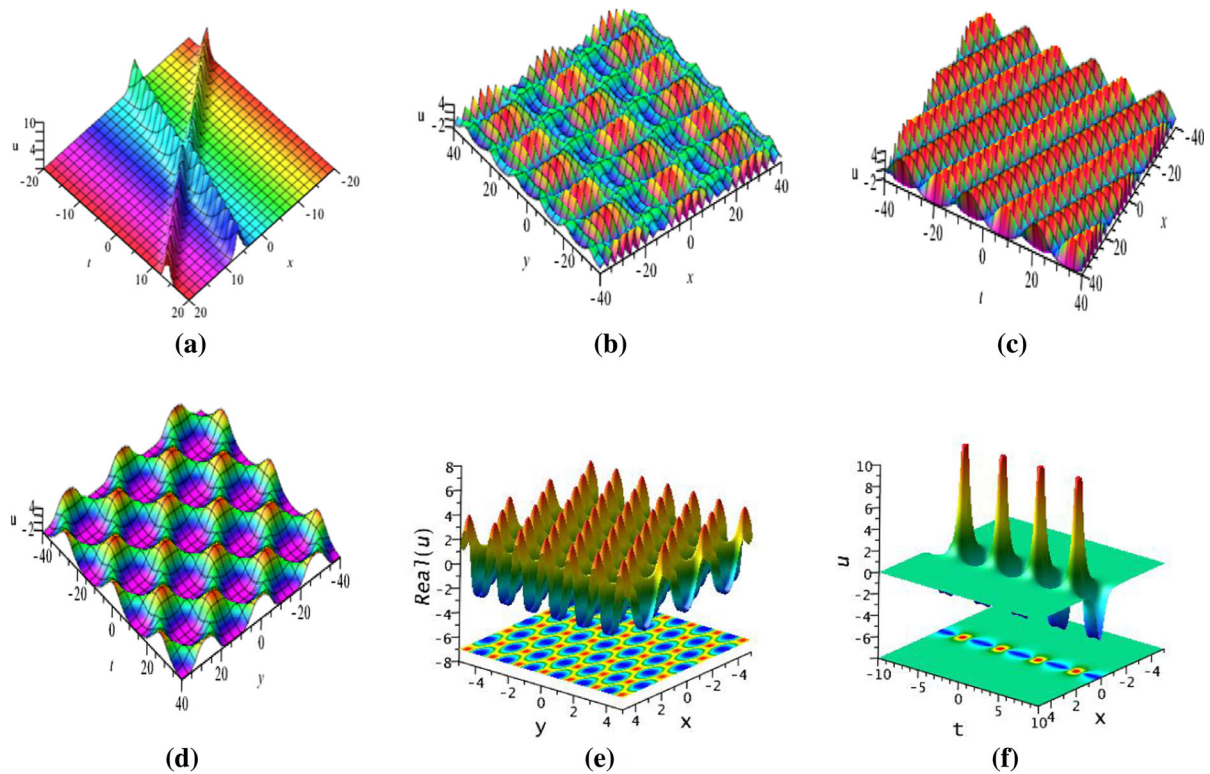


Fig. 2 **a** The two-soliton solutions of Eq. (1) for $\alpha = -0.005$, $\beta = 1$, $\gamma = 0.5$, $k_1 = k_2 = 2$, $p_1 = 1$, $p_2 = 2$, and $\sigma_1^0 = \sigma_2^0 = 0$ with $y = 0$. Breather wave solutions of Eq. (1) intended for:

$\alpha = 2$, $\beta = 4$, $\gamma = 1$, $k_1 = k_2^* = 2I$, $p_1 = -p_2 = 2$, $\sigma_1^0 = \sigma_2^0 = 0$, 3-D profile in various planes with: **b** $t = 0$; **c** $y = 0$; **d** $x = 0$, **e** Real part of double periodic wave via Eq. (21), **f** periodic line rogue waves via Eq. (22) (3-D upper and density lower)

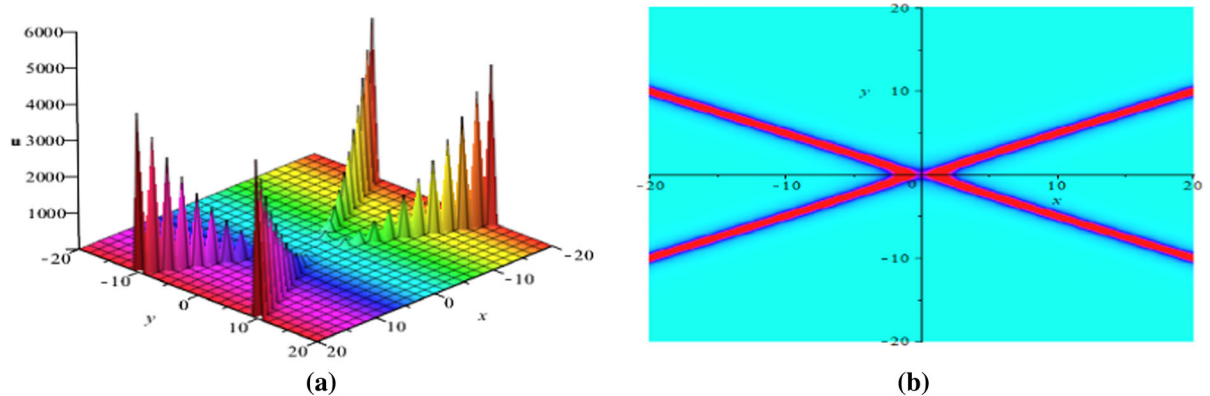


Fig. 3 Rogue shape solutions of Eq. (26) with picking appropriate parameters: $\alpha = -0.005$, $\beta = -1$, $\gamma = 0.5$, $p_1 = 2$, $p_2 = -2$: **a** 3-D plot and **b** density plot at $t = 0$

an interaction of three bell solitons and interact at the origin.

Besides, the values $\alpha = -0.005$, $\beta = 3$, $\gamma = 0.5$, $k_1 = k_2^* = I$, $k_3 = 3$, $p_1 = 0.5$, $p_2 = -0.5$, $p_3 = -1$

and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = 0$, then the expression [Eq. (27)] reduces to

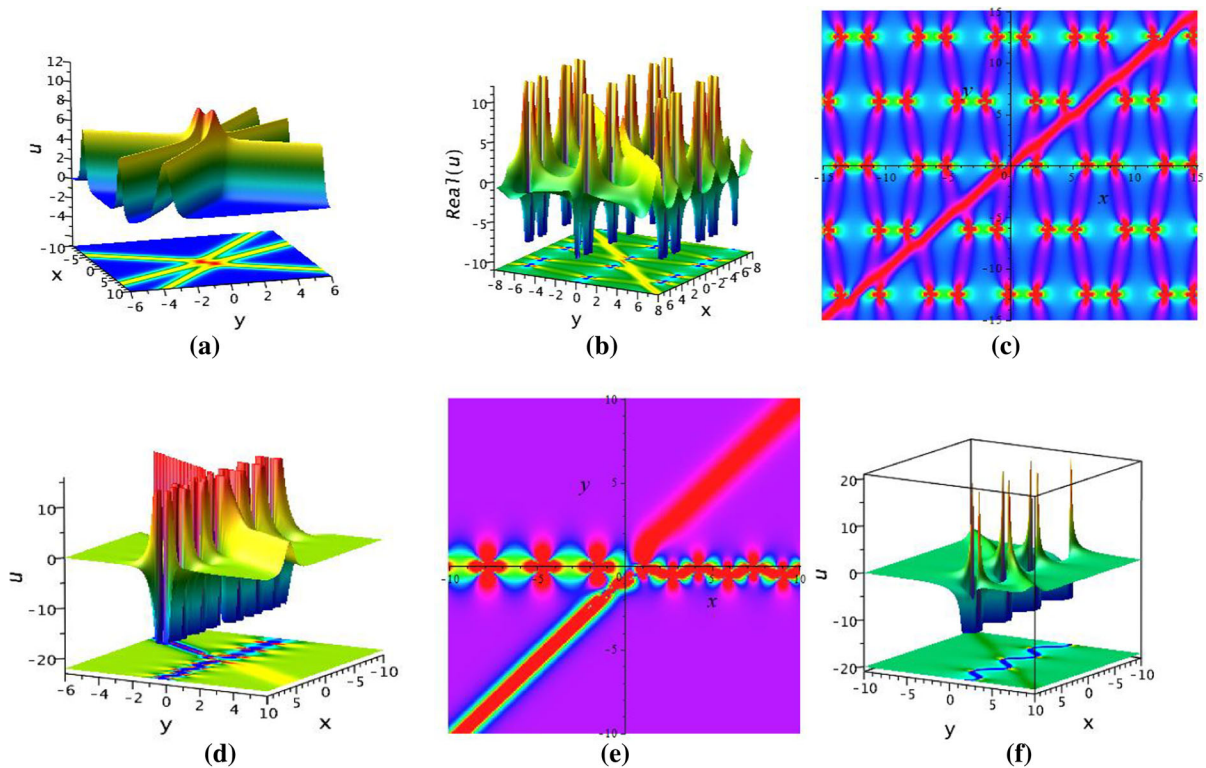


Fig. 4 **a** Interaction of three bell solitons via Eq. (27) (3-D upper, density plot lower), **b** interaction of double periodic line rogue with a bell soliton of the solution for Eq. (28) (3-D upper, density lower), **c** corresponding density plot of (b), **d** interaction

of a periodic line rogue with a bell soliton of the solution for Eq. (29) (3-D upper, density lower), **e** corresponding density plot of (d), **f** interaction of a periodic line rogue with a bell soliton of the solution for Eq. (30) (3-D upper, density lower)

$$f = 1 + 2\cos(x - 0.49t)\exp(0.5iy) + \exp(3x - 3y - 0.5427272727t) - 1.94\exp(iy) + 2r_1\cos(x - 0.49t + \phi_1)\exp(0.5iy + 3x - 3y - 0.5427272727t) + 1.94r_1^2\exp(iy + 3x - 3y - 0.5427272727t), \quad (28)$$

where $B_{13} = p_1 + Il_1 = r_1 \exp(-I\phi_1)$, $B_{23} = p_1 - Il_1 = r_1 \exp(I\phi_1)$ is illustrated in Fig. 4b, c. It is actually complex valued solution, and both real and imaginary parts present interaction of a bell soliton with double periodic rogue waves, in which bell soliton arises along diagonal. We provide also an extra density plot separately in Fig. 4c with different color combinations for better understanding.

For the values $\alpha = \beta = 1, \gamma = 0.5, k_1 = k_2^* = 2I, k_3 = 2, p_1 = p_2^* = I, p_3 = -1$ and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = 0$, the expression reduces to

$$f = 1 + 2\cos(2x)\exp(-2y) + \exp(2x - 2y - 1.333333333t) + \exp(-4y) - 2r_1\cos(2x + \phi_1)\exp(2x - 4y - 1.333333333t) + r_1^2\exp(2x - 6y - 1.333333333t), \quad (29)$$

where $B_{13} = p_1 + Il_1 = r_1 \exp(-I\phi_1)$, $B_{23} = p_1 - Il_1 = r_1 \exp(I\phi_1)$ which is depicted in Fig. 4d, e. It presents real-valued functions in which two bell solitons deformed into a single periodic rogue waves arise along x -axis that interact with the third bell soliton rises in the paradox. To realize the interaction directions and actual shape of the rogue wave, we provide the density plot separately (Fig. 4e).

A different type of such interaction is possible for slight changes in the parameters $\alpha = -0.005, \beta = 3, \gamma = 0.5, k_1 = k_2^* = I, k_3 = 2, p_1 = p_2^* = 0.5I, p_3 = -1$ and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = 0$, the expression [Eq. (27)] reduces to

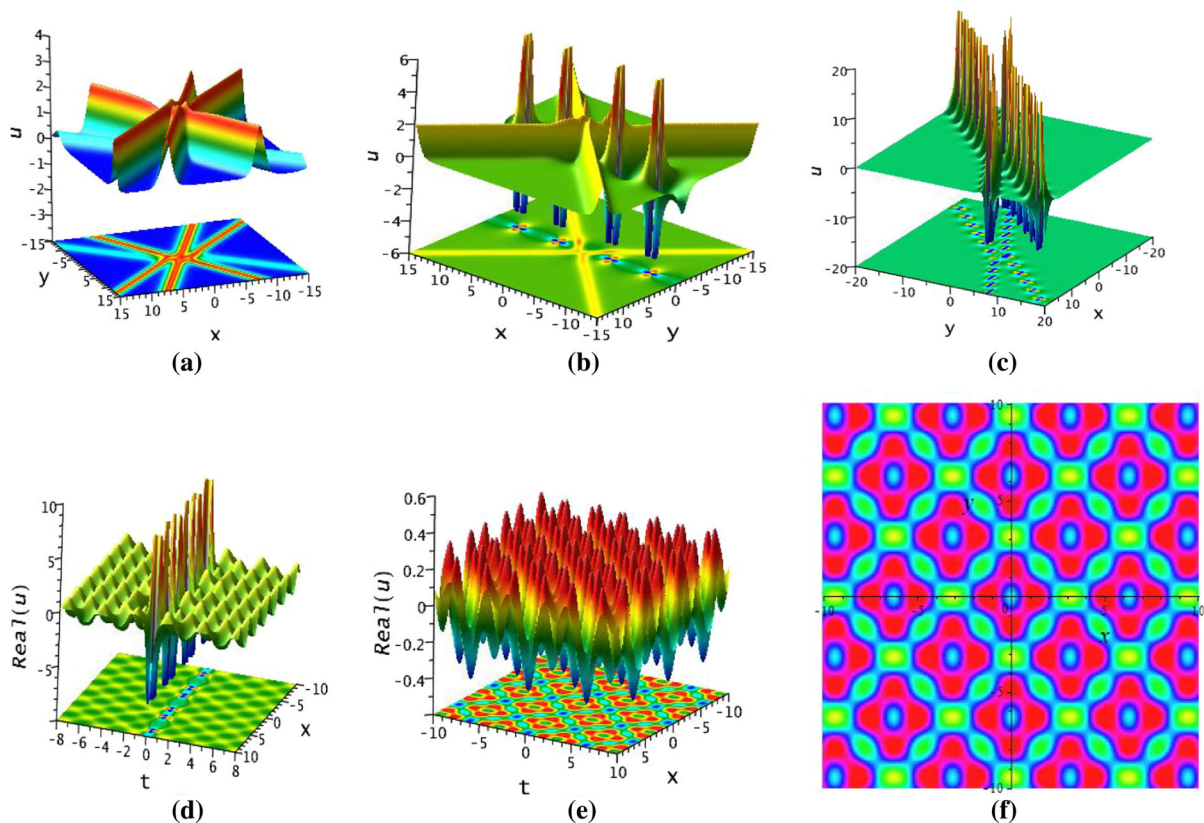


Fig. 5 **a** Interaction of four bell solitons via Eq. (31) (3-D upper, density plot lower), **b** interaction of two bell soliton with a periodic line rogue wave of the solution for Eq. (32) (3-D upper, density lower), **c** interaction of two periodic line rogue wave of the solution for Eq. (33) (3-D upper, density lower),

d interaction of a periodic line rogue with a periodic wave of the solution for Eq. (34) (3-D upper, density lower), **e** interaction of two periodic waves of the solution for Eq. (35) (3-D upper, density lower), **f** corresponding density plot of (e)

$$\begin{aligned}
 f = & 1 + 2 \cos(x + 0.51t) \exp(-0.5y) \\
 & + \exp(2x - 2y - 0.6633333334t) - 2.06 \exp(-y) \\
 & + 2r_1 \cos(x + 0.51t + \phi_1) \\
 & \exp(2x - 2.5y - 0.6633333334t) - 2.06r_1^2 \\
 & \exp(2x - 3y - 0.6633333334t),
 \end{aligned} \quad (30)$$

where $B_{13} = p_1 + Il_1 = r_1 \exp(-I\phi_1)$, $B_{23} = p_1 - Il_1 = r_1 \exp(I\phi_1)$ which is depicted in Fig. 4f. It is also real-valued but gives a similar with fascinating behaviors, see Fig. 4f.

4.3 Four solitons and their deformed structures into rogue, periodic, soliton with collisions

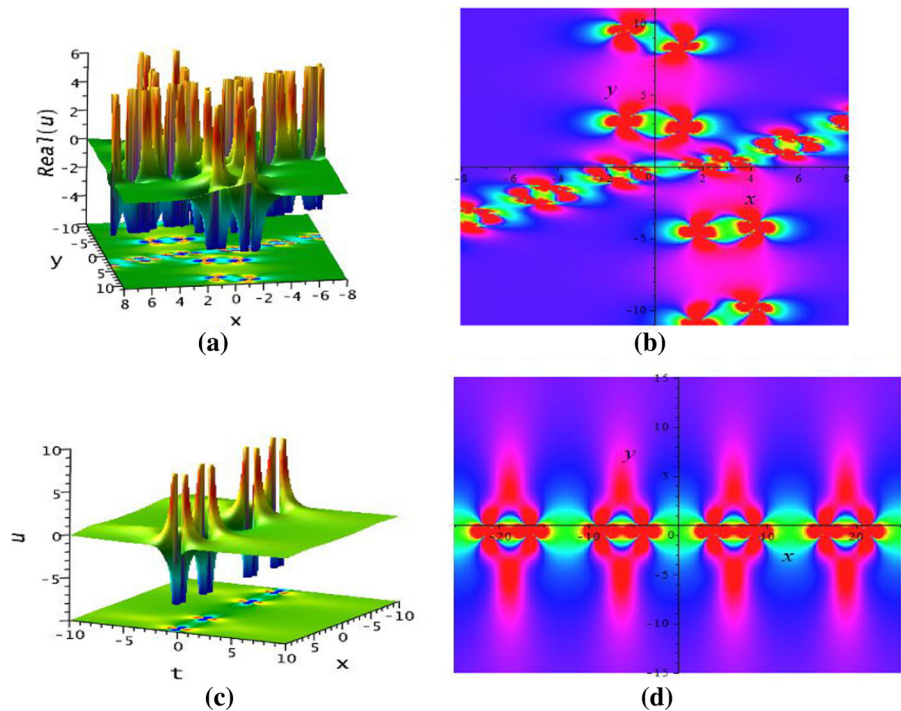
For $N = 4$, f_4 takes structure as

$$\begin{aligned}
 f_4 = & 1 + \exp \sigma_1 + \exp \sigma_2 + \exp \sigma_3 + \exp \sigma_4 \\
 & + B_{12} \exp(\sigma_1 + \sigma_2) + B_{13} \exp(\sigma_1 + \sigma_3) \\
 & + B_{14} \exp(\sigma_1 + \sigma_4) + B_{23} \exp(\sigma_2 + \sigma_3) \\
 & + B_{24} \exp(\sigma_2 + \sigma_4) + B_{34} \exp(\sigma_3 + \sigma_4) \\
 & + B_{12}B_{23}B_{13} \exp(\sigma_1 + \sigma_2 + \sigma_3) \\
 & + B_{12}B_{24}B_{14} \exp(\sigma_1 + \sigma_2 + \sigma_4) \\
 & + B_{13}B_{14}B_{34} \exp(\sigma_1 + \sigma_3 + \sigma_4) \\
 & + B_{23}B_{24}B_{34} \exp(\sigma_2 + \sigma_3 + \sigma_4) \\
 & + B_{12}B_{23}B_{13} \exp(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4),
 \end{aligned} \quad (31)$$

with $\sigma_i = k_i(x + p_i y + q_i t) + \sigma_i^0$, $i = 1, 2, 3, 4$. and q_i, B_{ij} where $i < j; i, j = 1, 2, 3, 4$ comes from Eq. (16), similar to Eq. (19).

For the values $\alpha = 1, \beta = 3, \gamma = 0.5, k_1 = -1, k_2 = k_3 = 2, k_4 = -2, p_1 = 1, p_2 = -1, p_3 = -0.5, p_4 = 0.5$

Fig. 6 **a** Collision of 2-pair of line rogue waves of the solution for Eq. (36) (3-D upper, density lower), **b** corresponding density plot of (a); **c** interaction of two periodic rogue waves in a line of the solution for Eq. (37) (3-D upper, density lower), **d** corresponding density plot of (c)



and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$ yields interaction of four bell solitons in Fig. 5a.

For the complex conjugate values $\alpha = 1, \beta = 3, \gamma = 0.5, k_1 = k_2^* = \sqrt{-1}, k_3 = 2, k_4 = -2, p_1 = p_2^* = \sqrt{-1}, p_3 = -1, p_4 = 1$. and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$ yields interaction of two bell solitons with a periodic rogue wave in Fig. 5b. Here, due to complex conjugate parametric values two bell soliton deformed into a periodic rogue wave and other two solitons remain their bell solitonic nature. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, in which

$$\begin{aligned} f = & 1 + 2\exp(-y)\cos x + \exp(-2y) \\ & + \exp(-2x - 2y + 1.333333333t) + 0.253886014 \\ & \exp(1.333333333t - 2x - 4y) \\ & + 2r_1 \exp(1.333333333t - 2x - 3y) \cos(x + g_1) + 2r_1 \\ & \exp(2x - 1.333333333t - 3y) \cos(x - g_1) \\ & + 5\exp(-4y) + 2(1.269430052) \exp(-5y) \cos x \\ & + 0.2538860104 \exp(2x - 4y - 1.333333333t) \\ & + 0.322290313 \exp(-6y), \end{aligned} \quad (32)$$

where $r_1 = 0.5038710256, g_1 = 1.042721878$.

The two bell line solitons interact with the periodic line rogue wave at the origin.

For the complex conjugate values of the parameters $\alpha = \gamma = 1, \beta = 3,$

$$p_2^* = \sqrt{-1}, p_3 = p_4^* = 2\sqrt{-1}.$$

and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$ yields interaction of two periodic rogue waves in Fig. 5c. Here, due to complex conjugate parametric values two pair of bell soliton deformed into a periodic rogue wave. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, where

$$\begin{aligned} f = & 1 + 2\exp(x - y) \cos(x + y) + \exp(2(x - y)) \\ & + 2\exp(x - 2y + 9t/5) \cos(x + 2y - 3t/5) \\ & + 2r_1 \exp(2x - 3y + 9t/5) \cos(y - 3t/5 + g_1) \\ & + 2r_2 \exp(3x - 4y + 9t/5) \cos(x + 2y - 3t/5 + g_2) \\ & + 2r_3 \exp(3x - 5y + 18t/5) \cos(x + y - g_3) \\ & + 2r_4 \exp(2x - 3y + 9t/5) \cos(2x + 3y - 3t/5 + g_4) \\ & + 17\exp(2x - 4y + 18t/5)/35 \\ & + 73\exp(4x - 6y + 18t/5)/86275, \end{aligned} \quad (33)$$

where $r_1 = \sqrt{10585}/145, g_1 = \tan^{-1}(96/37); r_2 = \sqrt{10585}/2465, g_1 = \tan^{-1}(1212/1261);$

$r_3 = \sqrt{10585}/5075$, $g_3 = \tan^{-1}(8724/607)$; $r_4 = 1/17$, $g_4 = \tan^{-1}(36/77)$. Figure 5c shows that each pair of bell solitons deformed into periodic rogue waves, due to four bell solitons presence in Eq. (31) yields two periodic line rogue waves that interact with each other at the origin.

The values $\alpha = 1, \gamma = 0.5, \beta = 3, k_1 = k_2^* = \sqrt{-1}, k_3 = k_4^* = 2\sqrt{-1}, p_1 = p_2^* = \sqrt{-1}, p_3 = -1, p_4 = 1$.

and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$ yields interaction of periodic rogue waves by a periodic wave described in Fig. 5d. Here, due to complex conjugate parametric values two pair of bell soliton deformed into a periodic rogue wave, and other two solitons reduce to a periodic wave. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, where

$$\begin{aligned} f = & 1 + 2 \exp(-y) \cos x + 2 \exp(-2Iy) \cos(2x + 4t) \\ & + \exp(-2y) + 2(3.769230769 \\ & + 1.846153846I) \exp(-y - 2Iy) \cos(x + 4t) \\ & + 2(0.6130332549 + 0.7900571042I) \\ & \exp(-2y - 2Iy) \cos(2x + 4t) + 13 \exp(-4Iy) \\ & + 2(0.2139737991 + 0.1048034934I) \\ & \exp(-y - 2Iy) \cos(3x + 4t) + 2(7.969432313 \\ & + 10.27074235I) \exp(-y - 2Iy) \cos x \\ & + (-3.228945929 + 12.59261323I) \exp(-2y - 4Iy). \end{aligned} \quad (34)$$

For the values $\alpha = 1, \gamma = 0.5, \beta = 3, k_1 = k_2^* = \sqrt{-1}, k_3 = k_4^* = 2\sqrt{-1}, p_1 = p_4 = 1, p_2 = p_3 = -1$, and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$, yields interaction of two periodic waves illustrated in Fig. 5e, f. Here, due to complex conjugate parametric values two pair of bell soliton deformed into a periodic wave. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, where

$$\begin{aligned} f = & 1 + 2 \exp(Iy) \cos(x - 4t) \\ & + 2 \exp(-2Iy) \cos(2x + 4t) - 5 \exp(2Iy) \\ & - (14/5) \exp(-Iy) \cos 3x \\ & - 6 \exp(-Iy) \cos x + 13 \exp(-4Iy) \\ & - 42 \cos 2x + (546/5) \\ & \exp(-3Iy) \cos x - (5733/5) \exp(-2Iy). \end{aligned} \quad (35)$$

The values $\alpha = -1, \gamma = 0.5, \beta = 3, k_1 = 1, k_2 = -1.1, k_3 = k_4^* = 1 + \sqrt{-1}, p_1 =$

$$p_2^* = \sqrt{-1}, p_3 = p_4^* = 2\sqrt{-1},$$

and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$, yields interaction of two pair of periodic rogue waves in Fig. 6a, b. Here, due to complex conjugate parametric values each pair of soliton deformed into a pair of periodic rogue waves. In this interaction solution, a pair of periodic rogue wave interact with the other pair of periodic rogue waves. Since the solution remains complex, its both real and imaginary parts exhibit the same natures. A separate density plot of the solution derived to understand the accurate shape and elastic situation before and after collision. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, where

$$\begin{aligned} f = & 1 + \exp(x + 1.333333333t + Iy) \\ & + \exp(-1.1x - 1.370716511t + 1.1Iy) \\ & + 2 \exp(x - 2y + 5t) \cos(x + 2y) + 3.120210438 \\ & \exp(-0.1x - 0.037383178t + 2.1Iy) \\ & + (0.2828108067 - 0.04140920585I) \\ & \exp(2x - 2y + 6.333333333t - I(x + y)) \\ & - (0.75602803 - 6.85492387I) \\ & \exp(-0.1x - 2y + 3.629283489t - I(x + 0.9y)) \\ & + (0.4398795039 + 1.964469291I) \\ & \exp(0.9x - 2y + 4.962616822t + I(x + 4.1y)) \\ & + (3.859430659 - 0.5278451737I) \\ & \exp(-0.1x - 2y + 3.629283489t + I(x + 3.1y)) \\ & + (0.2185521963 + 6.146668131I) \\ & \exp(0.9x - 2y + 4.962616822t - I(x - 0.1y)) \\ & + 0.166666667 \exp(2x - 4y + 10t) \\ & + (0.001796844008 + 0.007682880699I) \\ & \exp(3x - 4y + 11.333333333t + Iy) \\ & + (-0.6398502273 + 0.1673568962I) \\ & \exp(1.9x - 4y + 9.962616882t + 2.1Iy) \\ & + (0.1167501205 + 4.475861517I) \\ & \exp(0.9x - 4y + 8.629283489t + 1.1Iy) \\ & + (0.01395588614 + 0.1650403211I) \\ & \exp(2x - 2y + 6.333333333t + I(x + 3y)). \end{aligned} \quad (36)$$

For the choice of the complex conjugate values as $\alpha = 1, \gamma = 0.5, \beta = 3, k_1 = k_2^* = \sqrt{-1}, k_3 = k_4^* = \sqrt{-1}/2, p_1 = p_2^* = \sqrt{-1}, p_3 = p_4^* = \sqrt{-1}/2$.

and $\sigma_1^0 = \sigma_2^0 = \sigma_3^0 = \sigma_4^0 = 0$, the solution [Eq. (31)] yields two periodic rogue waves in a line

depicted in Fig. 6c, d. Here, due to complex conjugate parametric values two pair of bell soliton deformed into a periodic rogue wave, both of them propagate in a line symmetrically to each other. The symmetric situation can be clear from its density plot in Fig. 6d. The expression of the solution is $u = \frac{12\gamma}{\beta} [\ln f]_{xx}$, with

$$\begin{aligned} f = & 1 + 2 \exp(-y) \cos x + 2 \exp(-y/4) \\ & \cos(x/2 - 0.4285714286t) + \exp(-2y) \\ & + 0.3255813954 \exp(-5y/4) \\ & \cos(3x/2 - 0.4285714286t) \\ & + 3.14285714 \exp(-5y/4) \\ & \cos(x/2 + 0.4285714286t) \\ & + 2.285714286 \exp(-y/2) \\ & + 0.511627906 \exp(-9y/4) \\ & \cos(x/2 - 0.4285714286t) \\ & + 1.169435216 \exp(-3y/2) \cos x \\ & + 0.149578923 \exp(-5y/2). \end{aligned} \quad (37)$$

5 Results and discussion

Osman et al. [43] derived lump wave solution, which is found in our solution [Eq. (12)]. In Ref. [43], Osman also derived one-, two- and three-soliton solutions which are covered by our found N-soliton solution [Eq. (16)], and in addition, our solution can produce any number of solitons solution. Beside this, rogue-type breather waves in a line, rogue and single solitary wave solutions were found by Qin et al. [45] to the gCHKP model in a different way reducing variables, that are covered by our results [Eq. (22)] as periodic line rogue waves. Taking various complex conjugate values of free parameters in two-, three- and four-soliton solution in Eq. (16), we constructed more complex nonlinear dynamical behaviors in this investigation. Such as double periodic breather waves in Fig. 2b–d and bullet-like double periodic waves in Fig. 2e are visualized via Eq. (21), a different type of rogue shape solution via Eq. (26) is depicted in Fig. 3, which comes from two-soliton solution. The interaction of double periodic line rogue with a bell soliton via Eq. (28) in Fig. 4b, a periodic line rogue with a bell soliton via Eq. (29) in Fig. 4b, a periodic line rogue with a bell soliton via Eq. (30) in Fig. 4f are derived from three-soliton solution. From the four

soliton solution, we constructed interaction of two bell solutions with a periodic rogue wave depicted in Fig. 5b, two periodic line rogue wave via Eq. (33) visualized in Fig. 5c, periodic line rogue with a periodic wave via Eq. (34) displayed in Fig. 5d, two periodic waves illustrated in Fig. 5e. We also established colliding of two pair of periodic curvy rogue waves via Eq. (36) shown in Fig. 6a, b, two periodic rogue waves in a line solution via Eq. (37) depicted in Fig. 6c, d. Most of these dynamics are found for the first time in the literature.

6 Conclusions

We have successfully look over the (2 + 1)-D gCHKP model, that has claimed processing of collisions of exponentially confined edifices exact solutions. By exploitation the Hirota bilinear structure, we have acquired the lump-, breather-, solitary- as well as vast colliding-wave solutions among the solutions. Such colliding wave's solutions arises as interaction between bell soliton and double periodic rogue waves, rogue and bell soliton, rogue and two bell solitons, two rogues, rogue and periodic wave, double periodic waves, two pair of rogue waves as well as interaction of double periodic rogue waves. In addition, some coherent solutions are stated charming by a long limit. Discussion of the outcomes reveals to comprehend multifaceted corporal occurrences of the gCHKP equation with sufficient graphical diagrams. The achieved results disclosed that the imposed investigation process advocates a straight as well as simple methodical tackle to handle any nonlinear harm over and done with the bilinear method occurring in engineering and applied problems. From experiences of these studies, we also aim to analyze the chaotic behavior of the gCHKP model following the detection using chaos analysis in [51–53] in near future. Our future task will also be to draw dynamical behavior of variable coefficient gCHKP model in this similar procedure.

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Data availability The authors sketched the dynamical interactions of solitons and rogue with Maple. So, supported data are included inside this article and not taken from outside sources.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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